Friction Problems – Solution Strategy

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General Principles (problems on a plane)

1. Draw free-body-diagrams for all bodies in the problem
2. Write equilibrium equations for bodies.
   a. 2 force balance equations for slip
   b. 2 force balance equations and 1 moment balance equation for bodies that can tip or roll.
3. Write additional equations connecting the forces that act on the bodies from rods, cables, etc. These equations arise from:
   a. Principle of constant tension along a continuous cable.
   b. Equilibrium of frictionless pulleys.
   c. Newton’s third law.
4. Count the number of unknowns ($K$) in the problem.
5. Count the number of equations available ($N$) from steps 2 and 3.
6. Determine the difference $M=K-N$. If $M$ was negative, the modeling of the problem was incorrect. If $M$ was positive, we are $M$ equations short.
7. Write the friction force inequalities for all rough contacts. Say, there is $J$ number of inequalities.
8. Change $M$ number of inequalities into equalities.
9. Solve for the unknowns.
10. Check the validity of the remaining $J-M$ number of inequalities.
11. Accept solution if all the inequalities were satisfied in step-10. Else, discard the solution.
12. Go to step-8 and choose a different set of inequalities and change these into equalities.
Problem-1

A 50lb block of uniform density is on an incline. The coefficient of static friction between the block and incline is 0.3. Determine the maximum value of P for which the body remains in equilibrium. Is the impending motion by tipping or slipping up the incline?

**Tipping:**

- How many bodies? 1 (body A)
- How many rough surfaces? 1
  - Between A and floor, \( \mu \) known

**Body A**

- Weight of A: known
- How many unknown forces? 1
  - \( P(A1) = \text{force } P \)
- Motion: tip/roll (force and moment balance)
Additional equation for $P(A1)$: 0

Equilibrium equations for $A$: 3

\[
P(A1)\sin 60 + N(AFloor) - 50\cos 40 = 0 \\
P(A1)\cos 60 - F(AFloor) - 50\sin 40 = 0 \\
3 \times 50 \cos 40 + 2 \times 50 \sin 40 - 4 \times P(A1)\cos 60 - 6 \times P(A1)\sin 60 = 0
\]

Additional equations: 0
Friction inequalities changed to equalities: 0 out of 1

Find unknowns: $N(AFloor)$, $F(AFloor)$, $P(A1)$

(Three equations and three unknowns)

Check inequality:

\[F(AFloor) \leq 0.3N(AFloor)\]

Discard solution if inequality was violated.
Slipping:

- How many bodies? 1 (body A)
- How many rough surfaces? 1
  - Between A and floor, $\mu$ known

- Weight of A: known
- How many unknown forces? 1
  - $P(A1) = \text{force } P$
- Motion: slip (force balance)

- Additional equation for $P(A1)$: 0
Equilibrium equations for A: 2

\[ P(A1) \sin 60 + N(AFloor) - 50 \cos 40 = 0 \]
\[ P(A1) \cos 60 - F(AFloor) - 50 \sin 40 = 0 \]

Friction inequalities changed to equalities: 1 out of 1

\[ F(AFloor) = 0.3N(AFloor) \]

Find unknowns: N(AFloor), F(AFloor), P(A1)  
(Three equations and three unknowns)

Tip or Slip

Compare P(A1) values for tip and slip solutions; pick the smaller of the two solutions.
Problem-2

Blocks A and B weigh 20lb and 50lb, respectively. The blocks are connected by a continuous cable that passes over a frictionless pulley. The coefficient of static friction between the blocks is 0.2. The motion of the blocks is impending. What is the friction coefficient between block B and floor and what is the tension in the cable?

- How many bodies? 2 (bodies A and B)
- How many rough surfaces? 2
  - Between A and B, $\mu$ known
  - Between B and floor, $\mu$ unknown
- Figure-2b -

- **Body A**
  - Weight of A: known
  - How many unknown forces? 1
    - \( P(A1) = \text{force from cable} \)
  - Motion: slip (force balance)

- **Body B**
  - Weight of B: known
  - How many unknown forces? 1
    - \( P(B1) = \text{force from cable} \)
  - Motion: slip (force balance)

- **Figure-2c**

  - Additional equation connecting \( P(A1) \) and \( P(B1) \): 1
    - Continuous cable

\[ P(A1) = P(B1) \]

- **Figure-2d**

**Solution Strategy**

**Equilibrium Equations**

<table>
<thead>
<tr>
<th>For body A</th>
<th>For body B</th>
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<tbody>
<tr>
<td>2</td>
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Unknown forces: \( N(AB) \) \( F(AB) \) \( N(BFloor) \) \( F(BFloor) \) \( \mu(BFloor) \) \( P(A1) \) \( P(B1) \)

Change 2 out of 2 INEQUALITIES into EQUALITIES

- \( F(AB) = \mu(AB) N(AB) \)
- \( F(BFloor) = \mu(BFloor) N(BFloor) \)

VERIFY REMAINING INEQUALITIES
- Equilibrium equations for A: 2

\[ N(AB) - 20 \cos 25 = 0 \]
\[ P(A1) + F(AB) - 20 \sin 25 = 0 \]

- Equilibrium equations for B: 2

\[ N(BFloor) - N(AB) - 50 \cos 25 = 0 \]
\[ P(B1) + F(BFloor) - F(AB) - 50 \sin 25 = 0 \]

- Friction inequalities changed to equalities: 2 out of 2

\[ F(AB) = 0.2N(AB) \]
\[ F(BFloor) = \mu(BFloor)N(BFloor) \]

Find unknowns: \( N(AB), F(AB), N(BFloor), F(BFloor), \mu(BFloor), P(A1), P(B1) \).

*(Seven equations and seven unknowns)*
Problem-3

Block A weighs 20lb and block B weighs 50lb. The coefficient of static friction at all surfaces is 0.3. Find the force Q and the tension in the rod when motion of block B is impending.

Figure-3a

- How many bodies? 2 (bodies A and B)
- How many rough surfaces? 2
  - Between A and B, $\mu$ known
  - Between B and floor, $\mu$ known

Figure-3b

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- Body A
  - Weight of A: known
  - How many unknown forces? 1
    - $P(A1) =$ force from bar
  - Motion: slip (force balance)
- Body B
  - Weight of B: known
  - How many unknown forces? 1
    - $P(B1) =$ force Q
  - Motion: slip (force balance)

Additional equation connecting $P(A1)$ and $P(B1)$: 0

Equilibrium equations for A: 2

\[
N(AB) + P(A1) \sin 30 - 20 = 0 \\
F(AB) - P(A1) \cos 30 = 0
\]
- Equilibrium equations for B: 2

\[ N(BFloor) - N(AB) - 50 = 0 \]
\[ P(B1) - F(BFloor) - F(AB) = 0 \]

- Friction inequalities changed to equalities: 2 out of 2

\[ F(AB) = 0.3N(AB) \]
\[ F(BFloor) = 0.3N(BFloor) \]

Find unknowns: N(AB), F(AB), N(BFloor), F(BFloor), P(A1), P(B1).
(Six equations and six unknowns)
Problem-4

The coefficient of friction between the 50lb block A and the floor is 0.3. Find force Q for impending motion.

- How many bodies? 2
  - Block A
  - Pin f
- How many rough surfaces? 1
  - Between A and floor, $\mu$ known
- Figure-4b -

- **Body A**
  - Weight of A: **known**
  - How many unknown forces? **1**
    - \( P(A1) = \text{force from bar, } F(ef) \)
  - Motion: **slip** (force balance)

- **Body B**
  - Weight of B: **known** (weight of pin = 0)
  - How many unknown forces? **3**
    - \( P(B1) = \text{force from bar, } F(fe) \)
    - \( P(B2) = \text{force from bar, } F(fg) \)
    - \( P(B3) = \text{force } Q \)
  - Motion: **slip** (force balance)

- Figure-4c

- **Additional equation connecting** \( P(A1) \) **and** \( P(B1) **: 1**
  - Newton’s 3\(^{rd}\) Law

\[
P(A1) = P(B1)
\]
Equilibrium equations for A: 2

\[ N(A_{Floor}) + P(A1) \sin 25 - 50 = 0 \]
\[ F(A_{Floor}) - P(A1) \cos 25 = 0 \]

Equilibrium equations for B: 2

\[ P(B1) \cos 25 - P(B2) \cos 80 + P(B3) \cos 40 = 0 \]
\[ - P(B1) \sin 25 + P(B2) \sin 80 + P(B3) \sin 40 = 0 \]

Friction inequalities changed to equalities: 1 out of 1

\[ F(A_{Floor}) = 0.3N(A_{Floor}) \]

Find unknowns: \( N(A_{Floor}), F(A_{Floor}), P(A1), P(B1), P(B2), P(B3) \).
(Six equations and six unknowns)
Problem-5

The cylinders A and B weigh 50lb and 80lb, respectively. The coefficient of friction at the contact between A and B is 0.4. The coefficient of friction is 0.2 at all other contacts. Determine the force P for impending motion.

- How many bodies? 2 (bodies A and B)
- How many rough surfaces? 3
  - Between A and B, $\mu$ known
  - Between A and floor, $\mu$ known
  - Between B and floor, $\mu$ known
Body A
- Weight of A: known
- How many unknown forces? 1
  - P(A1) = applied force P
- Motion: tip/roll (force and moment balance)

Body B
- Weight of B: known
- How many unknown forces? 0
- Motion: tip/roll (force and moment balance)

Additional equation for P(A1): 0
Solution Strategy

Equilibrium Equations:

For body A: 3

For body B: 3

OK

Unknown forces: \( N(AB) \), \( F(AB) \), \( N(AFloor) \), \( F(AFloor) \), \( N(BFloor) \), \( F(BFloor) \), \( P(A1) \)

Change 1 out of 3 INEQUALITIES into EQUALITIES

\[
F(AB) = \mu N(AB) \\
F(AFloor) = \mu N(AFloor) \\
F(BFloor) = \mu N(BFloor)
\]

VERIFY REMAINING INEQUALITIES

Figure-5d

- Equilibrium equations for A: 3

\[
N(AFloor) + P(A1) \sin 50 - F(AB) - 50 \cos 20 = 0 \\
P(A1) \cos 50 + F(AFloor) - N(AB) - 50 \sin 20 = 0 \\
F(AFloor) - F(AB) = 0
\]

- Equilibrium equations for B: 3

\[
N(BFloor) + F(AB) - 80 \cos 20 = 0 \\
N(AB) + F(BFloor) - 80 \sin 20 = 0 \\
F(AB) - F(BFloor) = 0
\]

- Friction inequalities changed to equalities: 1 out of 3
  
  o Assumption-1

\[
F(AB) = 0.4N(AB)
\]

Find unknowns: \( N(AB) \), \( F(AB) \), \( N(Floor) \), \( F(AFloor) \), \( N(BFloor) \), \( F(BFloor) \), \( P(A1) \).

(Seven equations and seven unknowns)

Verify inequalities

\[
F(AFloor) \leq 0.2N(AFloor) \\
F(BFloor) \leq 0.2N(BFloor)
\]

Discard solution if inequalities were violated.
o **Assumption-2**

\[ F(AFloor) = 0.2N(AFloor) \]

Find unknowns: \( N(AB), F(AB), N(AFloor), F(AFloor), N(BFloor), F(BFloor), P(A1) \).

*(Seven equations and seven unknowns)*

Verify inequalities

\[ F(AB) \leq 0.4N(AB) \]
\[ F(BFloor) \leq 0.2N(BFloor) \]

Discard solution if inequalities were violated.

o **Assumption-3**

\[ F(BFloor) = 0.2N(BFloor) \]

Find unknowns: \( N(AB), F(AB), N(AFloor), F(AFloor), N(BFloor), F(BFloor), P(A1) \).

*(Seven equations and seven unknowns)*

 Verify inequalities

\[ F(AFloor) \leq 0.2N(AFloor) \]
\[ F(ABFloor) \leq 0.4N(AB) \]

Discard solution if inequalities were violated.

- Among the valid solutions pick the one with smallest \( P(A1) \).
### Problem-6

The blocks A, B, and C weigh 50lb, 40lb, and 60lb, respectively. Determine P for impending motion. Friction coefficient at all surfaces is 0.2.

#### Figure-6a

- **How many bodies?** 3 (bodies A, B, and C)
- **How many rough surfaces?** 3
  - Between A and B, μ known
  - Between A and floor, μ known
  - Between C and floor, μ known
Body A
- Weight of A: known
- How many unknown forces? 2
  - \( P(A1) = \text{applied force} \) \( P \)
  - \( P(A2) = \text{force from cable} \)
- Motion: slip (force balance)

Body B
- Weight of B: known
- How many unknown forces? 1
  - \( P(B1) = \text{force from cable} \)
- Motion: slip (force balance)

Body C
- Weight of C: known
- How many unknown forces? 1
  - \( P(C1) = \text{force from cable} \)
- Motion: slip (force balance)

Equations that relate unknown forces; in addition to equilibrium equations
In addition to equilibrium equations, how many equations can you write (BY USING NEWTON'S 3RD LAW, CONTINUOUS CABLE CONDITION etc.) that relate these unknown forces? 2

Figure-5c
Additional equations connecting \( P(A1), P(A2), P(B1), \) and \( P(C1) \): 2
- Continuous cable condition
  \[
P(A2) = P(B1)
\]
- Equilibrium of pulley
  \[
P(C1) = P(B1) + P(A2)
\]

### Solution Strategy

**Equilibrium Equations**

- For body A: 2
- For body B: 2
- For body C: 2

Unknown forces: \( N(AB), F(AB), N(AFloor), F(AFloor), N(CFloor), F(CFloor), P(A1), P(A2), P(B1), P(C1) \)

Change 2 out of 3 INEQUALITIES into EQUALITIES

- \( F(AB) = \mu N(AB) \)
- \( F(AFloor) = \mu N(AFloor) \)

\( F(CFloor) = \mu N(CFloor) \)

**VERIFY REMAINING INEQUALITIES**

Figure-5d

- **Equilibrium equations for A:** 2
  \[
P(A1) - P(A2) - F(AB) - F(AFloor) = 0
  
  N(AFloor) - N(AB) - 50 = 0
  \]

- **Equilibrium equations for B:** 2
  \[
  N(AB) - 40 = 0
  
  F(AB) - P(B1) = 0
  \]

- **Equilibrium equations for C:** 2
  \[
  N(CFloor) - 60 = 0
  
  P(C1) - F(CFloor) = 0
  \]
Friction inequalities changed to equalities: 2 out of 3

- Assumption-1

\[ F(A\text{Floor}) = 0.2N(A\text{Floor}) \]
\[ F(C\text{Floor}) = 0.2N(C\text{Floor}) \]

Find unknowns: N(AB), F(AB), N(AFloor), F(AFloor), N(CFloor), F(CFloor), P(A1), P(A2), P(B1), P(C1).

*(Ten equations and ten unknowns)*

Verify inequality
\[ F(AB) \leq 0.2N(AB) \]
Discard solution if inequality was violated.

- Assumption-2

\[ F(A\text{Floor}) = 0.2N(A\text{Floor}) \]
\[ F(AB) = 0.2N(AB) \]

Find unknowns: N(AB), F(AB), N(AFloor), F(AFloor), N(CFloor), F(CFloor), P(A1), P(A2), P(B1), P(C1).

*(Ten equations and ten unknowns)*

Verify inequality
\[ F(C\text{Floor}) \leq 0.2N(C\text{Floor}) \]
Discard solution if inequality was violated.

- Assumption-3

\[ F(C\text{Floor}) = 0.2N(C\text{Floor}) \]
\[ F(AB) = 0.2N(AB) \]

Find unknowns: N(AB), F(AB), N(AFloor), F(AFloor), N(CFloor), F(CFloor), P(A1), P(A2), P(B1), P(C1).

*(Ten equations and ten unknowns)*

Verify inequality
\[ F(A\text{Floor}) \leq 0.2N(A\text{Floor}) \]
Discard solution if inequality was violated.

- Among the valid solutions pick the one with smallest P(A1).
Problem-7

The blocks A, B, and C weigh 50lb, 40lb, and 60lb, respectively. Determine P for impending motion. Friction coefficient at all surfaces is 0.75.

- How many bodies? 3 (bodies are A, B, and C)
- How many rough surfaces? 3
  - Between A and floor, μ known
  - Between A and B, μ known
  - Between B and C, μ known
- Body A -
  - Weight of A: known
  - How many unknown forces? 0
  - Motion: slip (force balance)

- Body B -
  - Weight of B: known
  - How many unknown forces? 1
    - \( P(B1) \) = force \( P \)
  - Motion: slip (force balance)

- Body C -
  - Weight of C: known
  - How many unknown forces? 1
    - \( P(C1) \) = force from cable
  - Motion: slip (force balance)

Additional equations connecting \( P(B1) \) and \( P(C1) \): 0
Figure-5d

- Equilibrium equations for A: 2

\[ N(AFloor) - N(AB) - 50 \cos 33 = 0 \]
\[ F(AB) - F(AFloor) - 50 \sin 33 = 0 \]

- Equilibrium equations for B: 2

\[ N(AB) - N(BC) - 40 \cos 33 = 0 \]
\[ P(B1) - F(AB) - F(BC) - 40 \sin 33 = 0 \]

- Equilibrium equations for C: 2

\[ N(BC) - 60 \cos 33 = 0 \]
\[ F(BC) - P(C1) - 60 \sin 33 = 0 \]

- Friction inequalities changed to equalities: 2 out of 3
  - Assumption-1

\[ F(AFloor) = 0.75N(AFloor) \]
\[ F(AB) = 0.75N(AB) \]

Find unknowns: N(AFloor), F(AFloor), N(AB), F(AB), N(BC), F(BC), P(B1), P(C1).

*(Eight equations and eight unknowns)*

Verify inequality
\[ F(BC) \leq 0.75N(BC) \]

Discard solution if inequality was violated.

- **Assumption-2**

\[
F(\text{AFloor}) = 0.75N(\text{AFloor}) \\
F(BC) = 0.75N(BC)
\]

Find unknowns: \(N(\text{AFloor}), F(\text{AFloor}), N(\text{AB}), F(\text{AB}), N(\text{BC}), F(\text{BC}), P(\text{B1}), P(\text{C1})\).

*(Eight equations and eight unknowns)*

Verify inequality

\[ F(\text{AB}) \leq 0.75N(\text{AB}) \]

Discard solution if inequality was violated.

- **Assumption-3**

\[
F(BC) = 0.75N(BC) \\
F(AB) = 0.75N(AB)
\]

Find unknowns: \(N(\text{AFloor}), F(\text{AFloor}), N(\text{AB}), F(\text{AB}), N(\text{BC}), F(\text{BC}), P(\text{B1}), P(\text{C1})\).

*(Eight equations and eight unknowns)*

Verify inequality

\[ F(\text{AFloor}) \leq 0.75N(\text{AFloor}) \]

Discard solution if inequality was violated.

- *Among the valid solutions pick the one with smallest \(P(A1)\).*