Singularity Functions

Properties-1

For $n > 0$

$$< x - x_0 >^n = (x - x_0)^n ; x \geq x_0$$

$$< x - x_0 >^n = 0 ; x < x_0$$

For $n = 0$

$$< x - x_0 >^0 = 1 ; x \geq x_0$$

$$< x - x_0 >^0 = 0 ; x < x_0$$

Properties-2

$$\int < x - x_0 >^n \, dx = \frac{1}{n+1} < x - x_0 >^{n+1}; n \geq 0$$

$$\int < x - x_0 >^n \, dx = < x - x_0 >^{n+1}; n < 0$$

Basic Singularity Functions

Point Load P (+ is up)

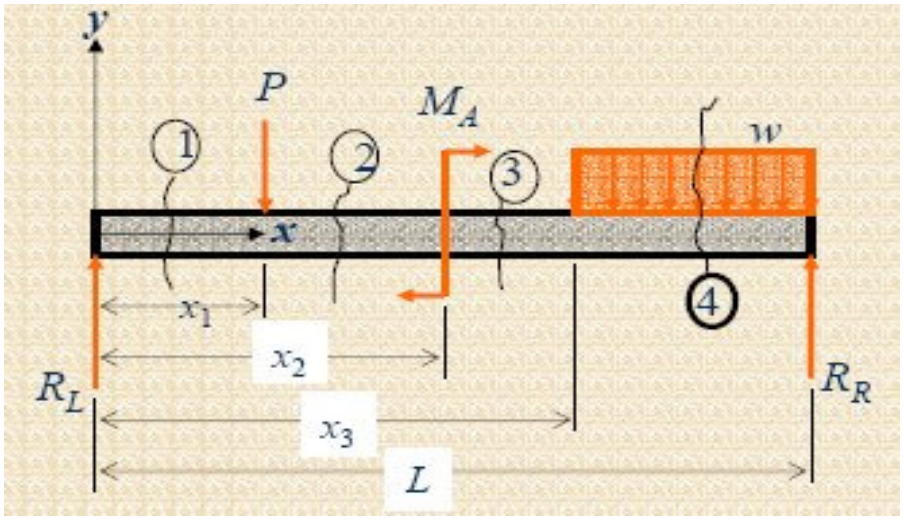
$$-P\langle x - x_1 \rangle^{-1}$$

Couple M_A (+ is CW)

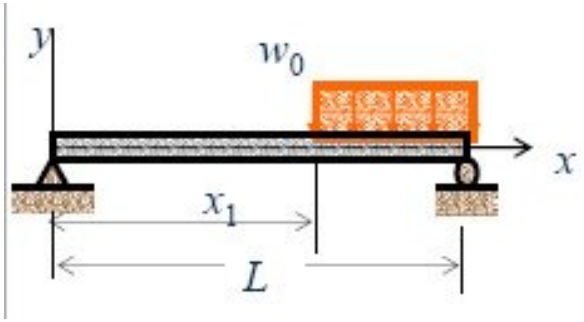
$$M_A\langle x - x_2 \rangle^{-2}$$

Distributed Load w (+ is up)

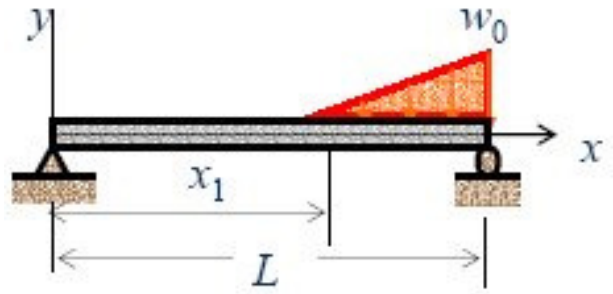
$$-w\langle x - x_3 \rangle^{0}$$



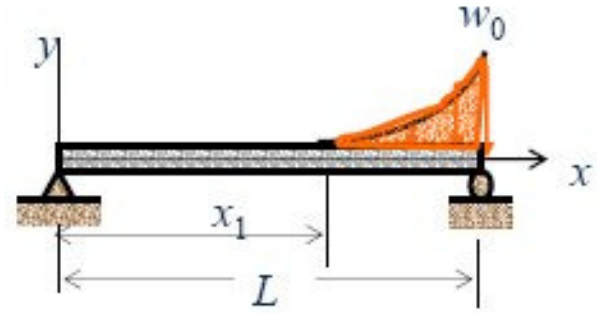
Distributed Loads -1



$$-w_0 < x - x_1 >^0$$

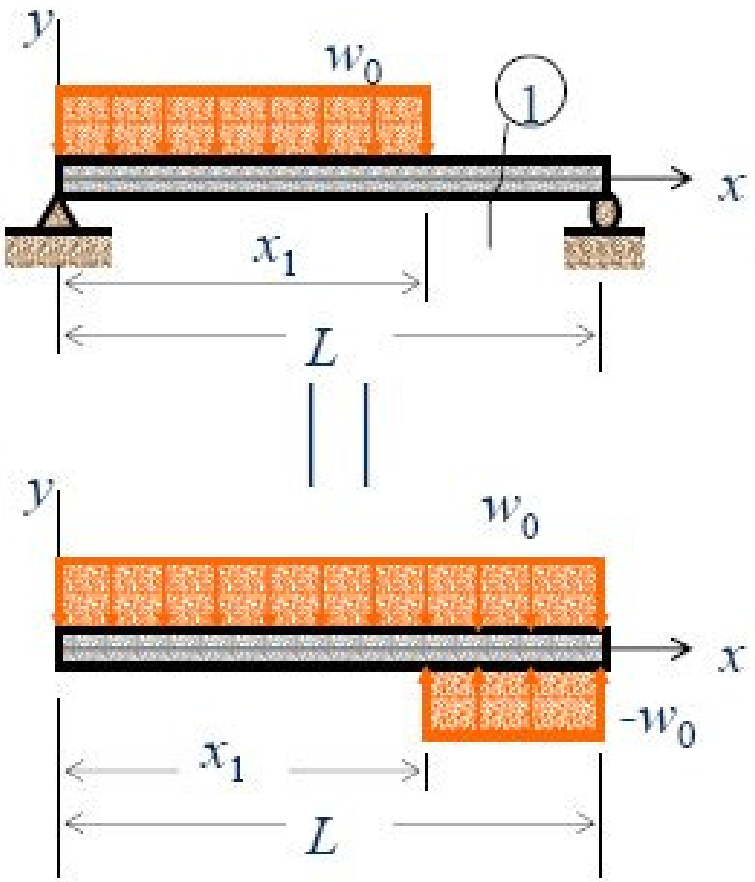


$$-\frac{w_0}{(L - x_1)} < x - x_1 >^1$$



$$-\frac{w_0}{(L - x_1)^2} < x - x_1 >^2$$

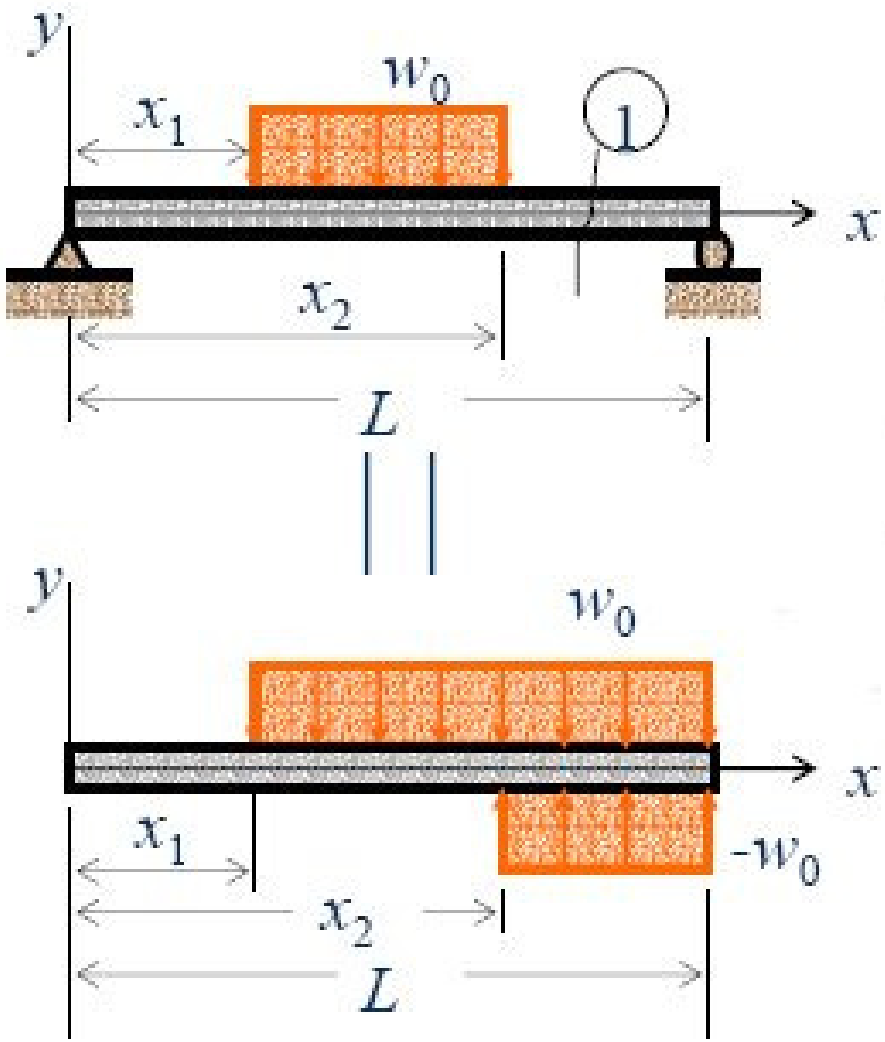
Distributed Loads -2



$$-w_0 < x-0 >^0$$

$$+w_0 < x-x_1 >^0$$

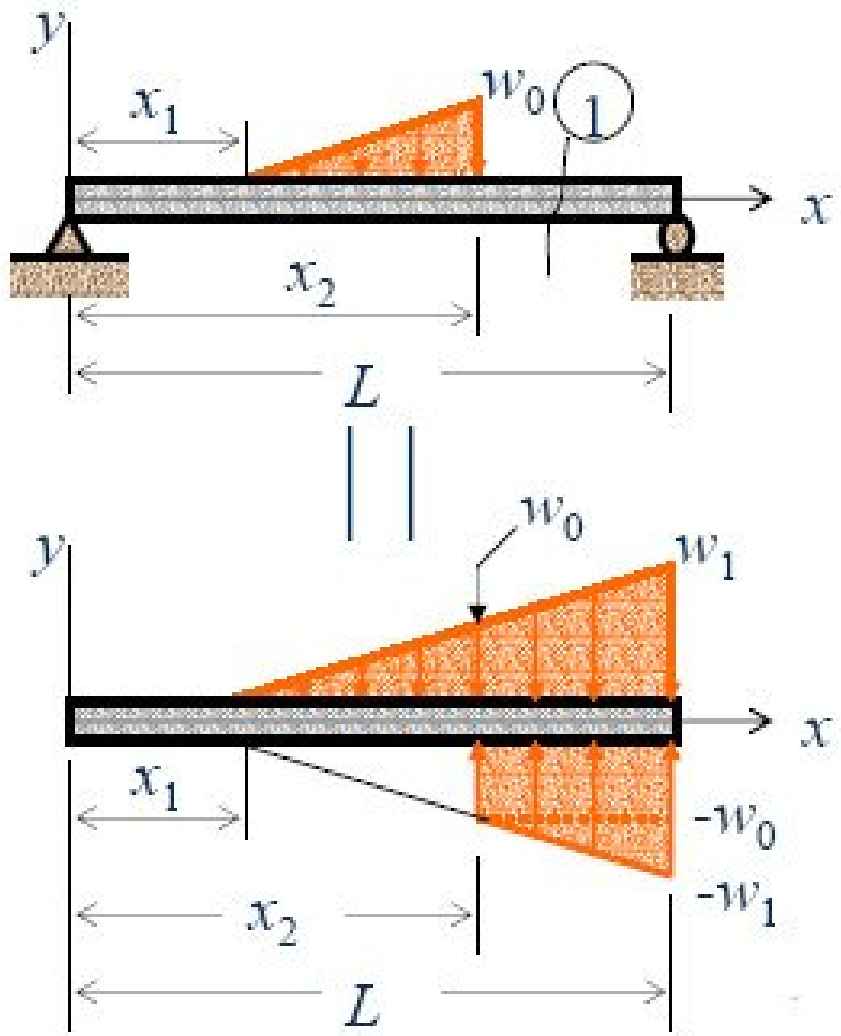
Distributed Loads -3



$$-w_0 < x - x_1 >^0$$

$$+w_0 < x - x_2 >^0$$

Distributed Loads -4



$$-\frac{w_0}{(x_2 - x_1)} < x - x_1 >^1$$

$$+\frac{w_0}{(x_2 - x_1)} < x - x_2 >^1$$

$$+ w_0 < x - x_2 >^0$$

Shear and Moment

- Loading = Applied loading + Support Reactions

  w(x) = sum of all singularity functions

- Shear = **V(x)** = $\int w(x)dx$

- Moment = **M(x)** = $\int V(x)dx$

(origin at the left end of the beam)

Beam Slope

- Simply Supported and Clamped Beams

  (origin at left end of the beam, clamp is assumed to be at left end)

$$\theta(x) = \frac{1}{EI}\int M(x)dx + C_1$$

Beam Deflection-1

- Simply Supported and Clamped Beams

$$y(x) = \int \theta(x)dx + C_1 x + C_2 = d(x) + C_1 x + C_2$$

Beam Deflection-2

- Simply Supported Beams
  (left support at x=L, right support at x=R)

$$C_1 L + C_2 = -d(L)\,;\; C_1 R + C_2 = -d(R)$$

- Clamped Beams (origin at clamp on the left end of beam)

$$C_1 = 0 \quad ; \quad C_2 = 0$$

Example-1

$$w(x) = -w\langle x-0\rangle^{0} + w\langle x-\frac{L}{2}\rangle^{0} + \frac{3wL}{8}\langle x-L\rangle^{-1} + \frac{wL^{2}}{2}\langle x-\frac{3L}{2}\rangle^{-2} + \frac{wL}{8}\langle x-2L\rangle^{-1}$$

$$V(x) = -w\langle x-0\rangle^{1} + w\langle x-\frac{L}{2}\rangle^{1} + \frac{3wL}{8}\langle x-L\rangle^{0} + \frac{wL^{2}}{2}\langle x-\frac{3L}{2}\rangle^{-1} + \frac{wL}{8}\langle x-2L\rangle^{0}$$

$$M(x) = -\frac{w}{2}\langle x-0\rangle^{2} + \frac{w}{2}\langle x-\frac{L}{2}\rangle^{2} + \frac{3wL}{8}\langle x-L\rangle^{1} + \frac{wL^{2}}{2}\langle x-\frac{3L}{2}\rangle^{0} + \frac{wL}{8}\langle x-2L\rangle^{1}$$

$$EI\theta(x) = -\frac{w}{6}\langle x-0\rangle^{3} + \frac{w}{6}\langle x-\frac{L}{2}\rangle^{3} + \frac{3wL}{16}\langle x-L\rangle^{2} + \frac{wL^{2}}{2}\langle x-\frac{3L}{2}\rangle^{1} + \frac{wL}{16}\langle x-2L\rangle^{2} + C_1$$

$$EIy(x) = -\frac{w}{24}\langle x-0\rangle^{4} + \frac{w}{24}\langle x-\frac{L}{2}\rangle^{4} + \frac{wL}{16}\langle x-L\rangle^{3} + \frac{wL^{2}}{4}\langle x-\frac{3L}{2}\rangle^{2} + \frac{wL}{48}\langle x-2L\rangle^{3} + C_1 x + C_2$$

$$C_1 L + C_2 = \frac{5}{128}wL^{4} \quad ; \quad 2C_1 L + C_2 = \frac{127}{384}wL^{4} \quad ; \quad C_1 = \frac{7}{24}wL^{3} \quad ; \quad C_2 = -\frac{97}{384}wL^{4}$$

Example-2

$$w(x) = -wL\langle x-0\rangle^{-1} + \frac{7wL^2}{2}\langle x-0\rangle^{-2} - w\langle x-0\rangle^{0} + w\langle x-L\rangle^{0} + 2wL\langle x-2L\rangle^{-1}$$

$$V(x) = -wL\langle x-0\rangle^{0} + \frac{7wL^2}{2}\langle x-0\rangle^{-1} - w\langle x-0\rangle^{1} + w\langle x-L\rangle^{1} + 2wL\langle x-2L\rangle^{0}$$

$$M(x) = -wL\langle x-0\rangle^{1} + \frac{7wL^2}{2}\langle x-0\rangle^{0} - \frac{w}{2}\langle x-0\rangle^{2} + \frac{w}{2}\langle x-L\rangle^{2} + 2wL\langle x-2L\rangle^{1}$$

$$EI\theta(x) = -\frac{wL}{2}\langle x-0\rangle^{2} + \frac{7wL^2}{2}\langle x-0\rangle^{1} - \frac{w}{6}\langle x-0\rangle^{3} + \frac{w}{6}\langle x-L\rangle^{3} + wL\langle x-2L\rangle^{2}$$

$$EIy(x) = -\frac{wL}{6}\langle x-0\rangle^{3} + \frac{7wL^2}{4}\langle x-0\rangle^{2} - \frac{w}{24}\langle x-0\rangle^{4} + \frac{w}{24}\langle x-L\rangle^{4} + \frac{wL}{3}\langle x-2L\rangle^{3}$$