# Stress at a Point

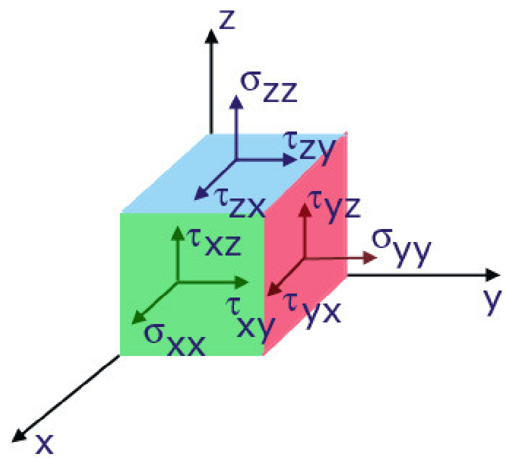


Figure-1

Rectangular Cuboid:
The rectangular cuboid, in the figure above, is of vanishing size or is a point. We have assumed the point to have that shape so that we can use a rectangular Cartesian coordinate system to conveniently analyze the 3-dimensional stress at the point.

In your advanced courses on Mechanics of Materials, you will see that points are given other shapes to analyze 3-dimensional stress in other orthogonal coordinate systems, such as cylindrical, spherical, etc.

Outward Normal:
The cuboid has six surfaces and the perpendiculars drawn on these six surfaces are the *normals.* The six outward normals are arrowheads pointing outward from the cuboid.

Surface Names:
The pointing directions of the arrowheads of the outward normals are used in naming the surfaces. For example, the arrowhead on the green surface points toward *+x* direction.

- green surface: +x
- red surface: +y
- blue surface: +z
- three hidden surfaces: -x, -y, and –z

Stress Symbols:
Normal stresses are denoted by $\sigma$ and tangential (shear) stresses are denoted by $\tau$.

Stress Subscripts:
The first subscript defines the surface on which the stress is acting.

- $\sigma_{xx}, \tau_{xy}, \tau_{xz}$ act on +x and –x surfaces.
- $\sigma_{yy}, \tau_{yx}, \tau_{yz}$ act on +y and –y surfaces.
- $\sigma_{zz}, \tau_{zy}, \tau_{zx}$ act on +z and –z surfaces.

The second subscript defines the direction along which the stress is acting.

- $\sigma_{xx}, \tau_{yx}, \tau_{zx}$ act along +x and –x directions.
- $\sigma_{yy}, \tau_{xy}, \tau_{zy}$ act along +y and –y directions.
- $\sigma_{zz}, \tau_{yz}, \tau_{xz}$ act along +z and –z directions.

Stress Directions:

- On the +*x*, +*y*, and +*z* surfaces: The stresses that act along the ***positive*** coordinate directions are given a ***plus*** sign. The stresses that act along the ***negative*** coordinate directions are given a ***negative*** sign.
- On the -*x*, -*y*, and -*z* surfaces: The stresses that act along the ***negative*** coordinate directions are given a ***plus*** sign. The stresses that act along the ***positive*** coordinate directions are given a ***negative*** sign.

Plane Stress:
$\sigma_{zz} = \tau_{xz} = \tau_{zx} = \tau_{yz} = \tau_{zy} = 0$

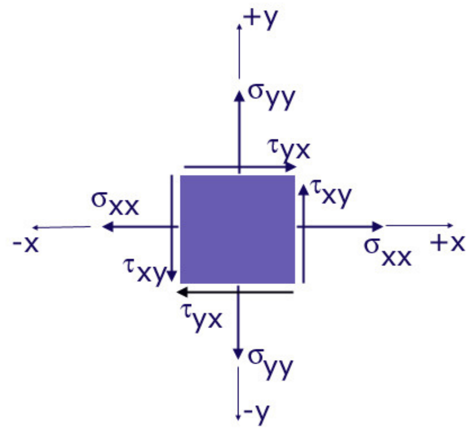


Figure-2

The plane stress condition is shown in the figure above. It is easily seen that the net force on the element is zero. For net moment on the element to be zero, we must have

$$\tau_{xy} = \tau_{yx}$$

Stress Transformation:

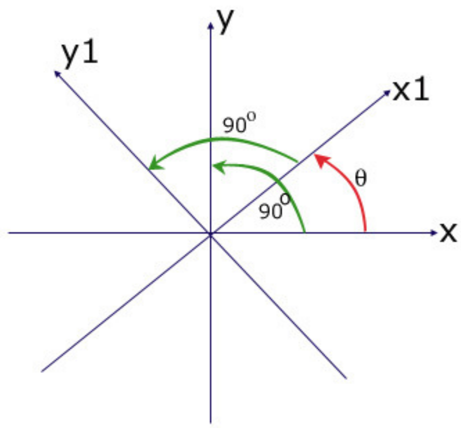


Figure-3

The ultimate goal of determining the stress at a point, either experimentally or theoretically, is to design the structural member that bears that stress and is to ensure that the structural member will not fail due to the stress. As the choice of the xy-coordinate system is totally arbitrary, the stress magnitudes in this coordinate system will not show the stress on a welding or a joint passing through that point, and will not show the maximum stresses at this point. Therefore we need a set of formula to determine the stress in another rotated coordinate system x1-y1.

Angle Convention:
x to y is 90 degrees counter-clockwise.
x1 to y1 is 90 degrees counter-clockwise.
x to x1 is $\theta$ counter-clockwise.

Stress Transformation Formulas:

$$\sigma_{x1x1} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \qquad Eq.(1)$$

$$\sigma_{y1y1} = \frac{\sigma_{xx} + \sigma_{yy}}{2} - \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \qquad Eq.(2)$$

$$\tau_{x1y1} = -\frac{\sigma_{xx} - \sigma_{yy}}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \qquad Eq.(3)$$

Observation:

$$\sigma_x + \sigma_y = \sigma_{x1x1} + \sigma_{y1y1} \qquad Eq.(4)$$

Example-1

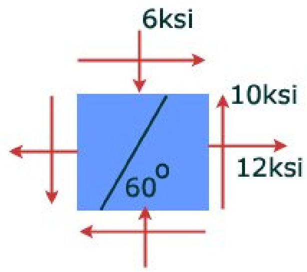


Figure-4

To find the stresses on the inclined surface, we rotate the horizontal-vertical, xy-system through an angle of 150°, so that the x1 axis becomes perpendicular to the inclined surface. Therefore,

$$\sigma_x = 12ksi \quad \sigma_y = -6ksi \quad \tau_{xy} = 10ksi \quad \theta = 150^o$$

From the stress-transformation equations (1) and (3)

$$\sigma_{x1x1} = \frac{12+(-6)}{2} + \frac{12-(-6)}{2} cos300^o + 10\, sin300^o = -1.16ksi \quad Eq.(5)$$

$$\tau_{x1y1} = -\frac{12-(-6)}{2} sin300^o + 10\, cos300^o = 12.8ksi \quad Eq.(6)$$

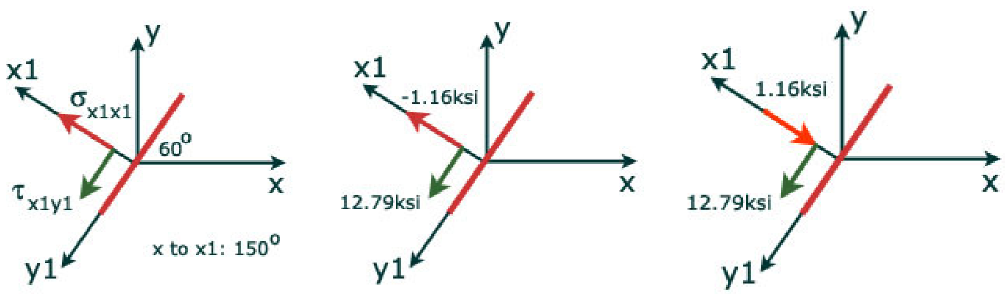


Figure-5(a) Figure-5(b) Figure-5(c)

The orientation of the axes system and the stresses are shown in the diagrams above. In Fig. 5(b), the normal stress has a negative sign, therefore we flipped it direction in Fig. 5(c).

Another way to solve this problem is to rotate the xy-system through an angle of 60° so that the y1-axis becomes perpendicular to the inclined surface.

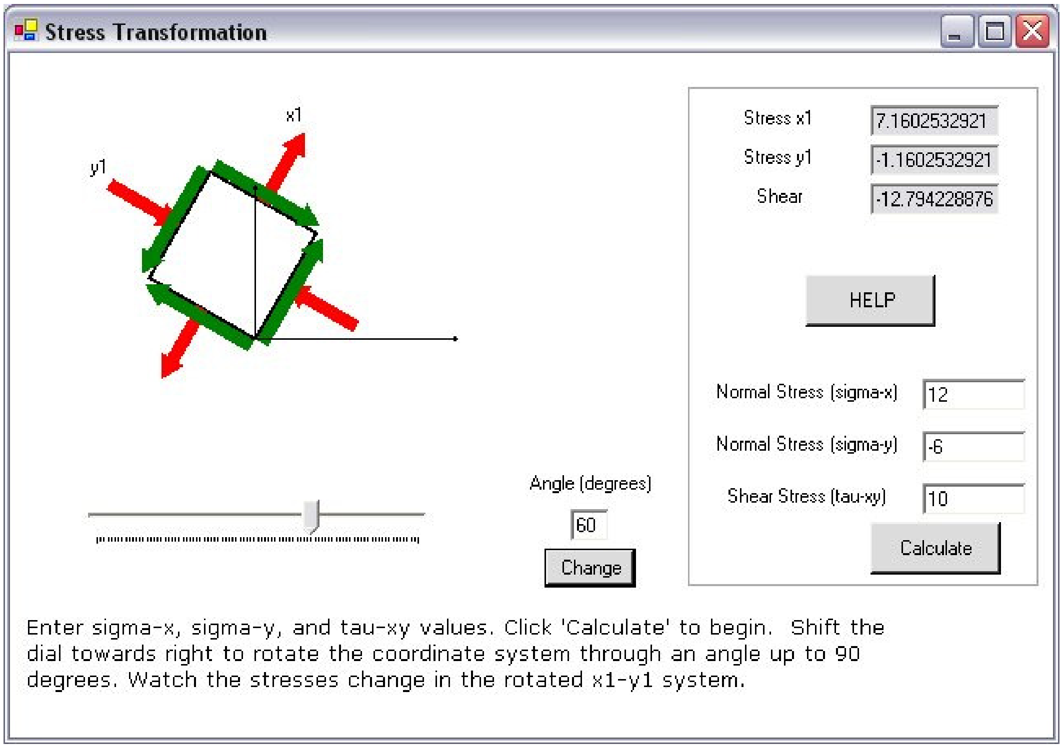


Figure-6

<u>Sticky Issue</u>

Why does Eq.(6) give a positive shear value of 12.8ksi whereas the software output in Fig.(6) shows a negative shear value of -12.8ksi?

For the calculation in Eq.(6) we made x1 axis perpendicular to the inclined surface. This gives us the stress directions of Fig. (7a).

For the calculation of Fig. (6), in the software, we made the y1 axis perpendicular to the inclined surface. This gives us the stress directions of Fig. 7(b).

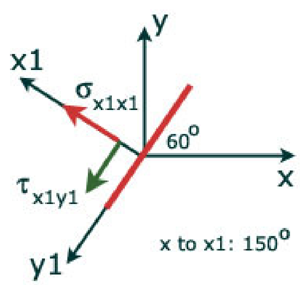


Figure-7(a)

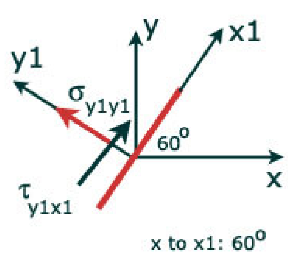


Figure-7(b)

By comparing Fig. 7(a) and 7(b), we can see that the direction of shear has switched.

Example-2

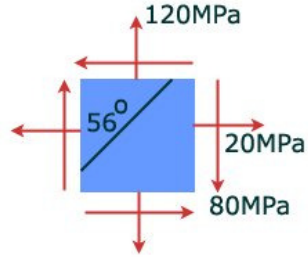


Figure-8

To find the stresses on the inclined surface, we rotate the horizontal-vertical, xy-system through an angle of 124°, so that the x1 axis becomes perpendicular to the inclined surface. Therefore,

$$\sigma_x = 20MPa \quad \sigma_y = 120MPa \quad \tau_{xy} = -80MPa \quad \theta = 124^o$$

From the stress-transformation equations (1) and (3)

$$\sigma_{x1x1} = \frac{20+120}{2} + \frac{12-120}{2} cos248^o + (-80) sin248^o = 163MPa$$

$$\tau_{x1y1} = -\frac{12-120}{2} sin248^o + (-80) cos248^o = -16.4MPa$$

Another way to solve this problem is to rotate the xy-system through an angle of 34° so that the y1-axis becomes perpendicular to the inclined surface.

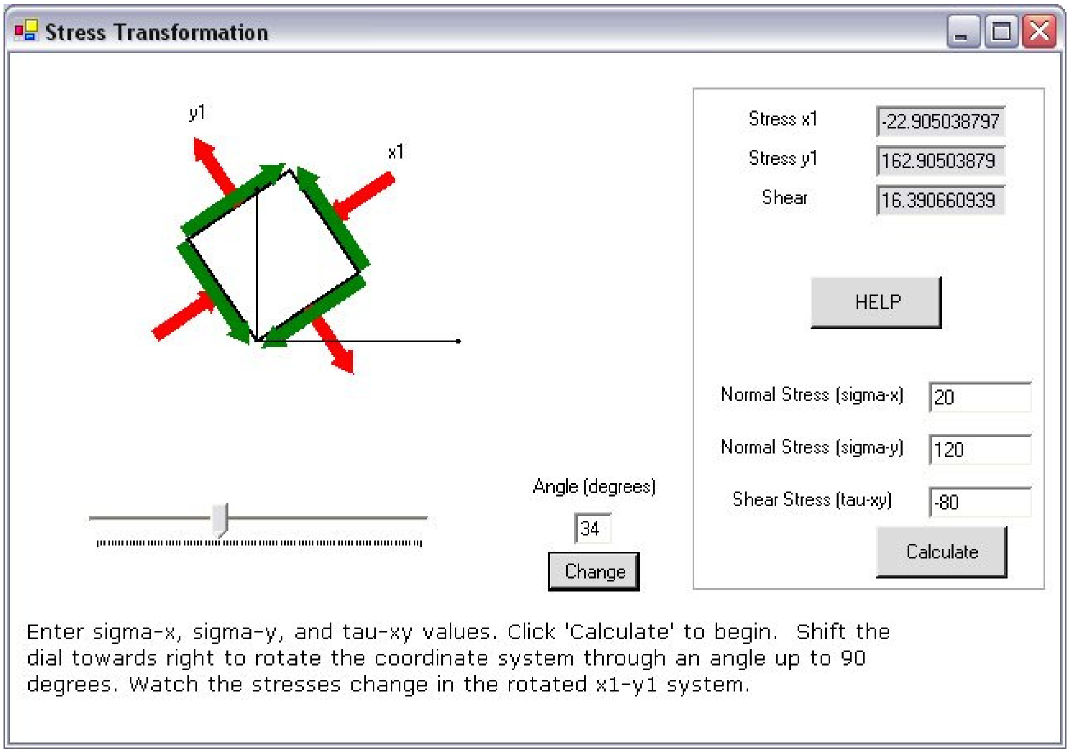


Figure-9

Example-3

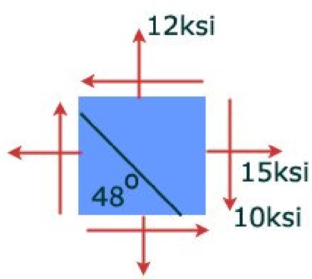


Figure-10

To find the stresses on the inclined surface, we rotate the horizontal-vertical, xy-system through an angle of 42°, so that the x1 axis becomes perpendicular to the inclined surface. Therefore,

$$\sigma_x = 15ksi \quad \sigma_y = 12ksi \quad \tau_{xy} = -10ksi \quad \theta = 222^o$$

From the stress-transformation equations (1) and (3)

$$\sigma_{x1x1} = \frac{15+12}{2} + \frac{15-12}{2} cos\,444^o + (-10) sin\,444^o = 3.71ksi$$

$$\tau_{x1y1} = -\frac{15-12}{2} sin\,444^o + (-10) cos\,444^o = -2.54ksi$$

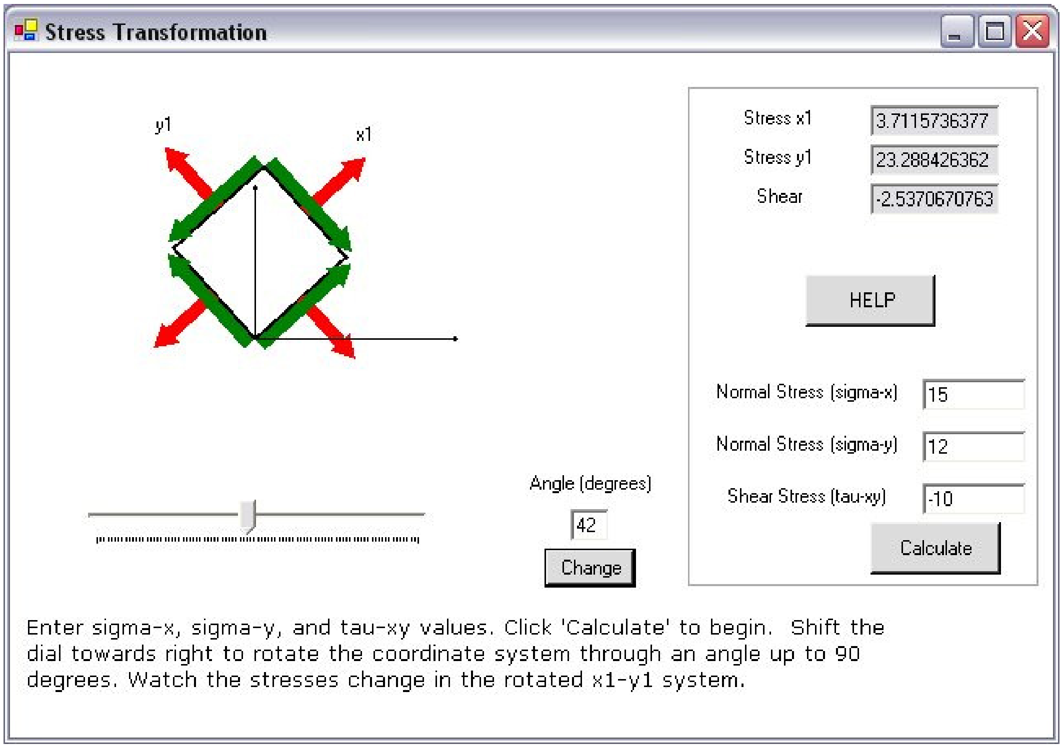


Figure-11

<u>Example-4</u>

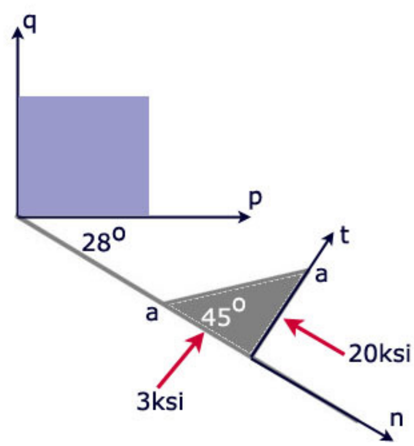


Figure-12

At a point in a stressed body, the stresses in the nt-system are shown in Fig.12.

a. Calculate the stresses on plane aa
b. Calculate the stresses in pq-system

*<u>Part-a</u>*

Imagine that n=x and y=t. When the xy-system (nt-system) is turned through an angle of $45^o$, the y1-axis becomes perpendicular to the aa plane.

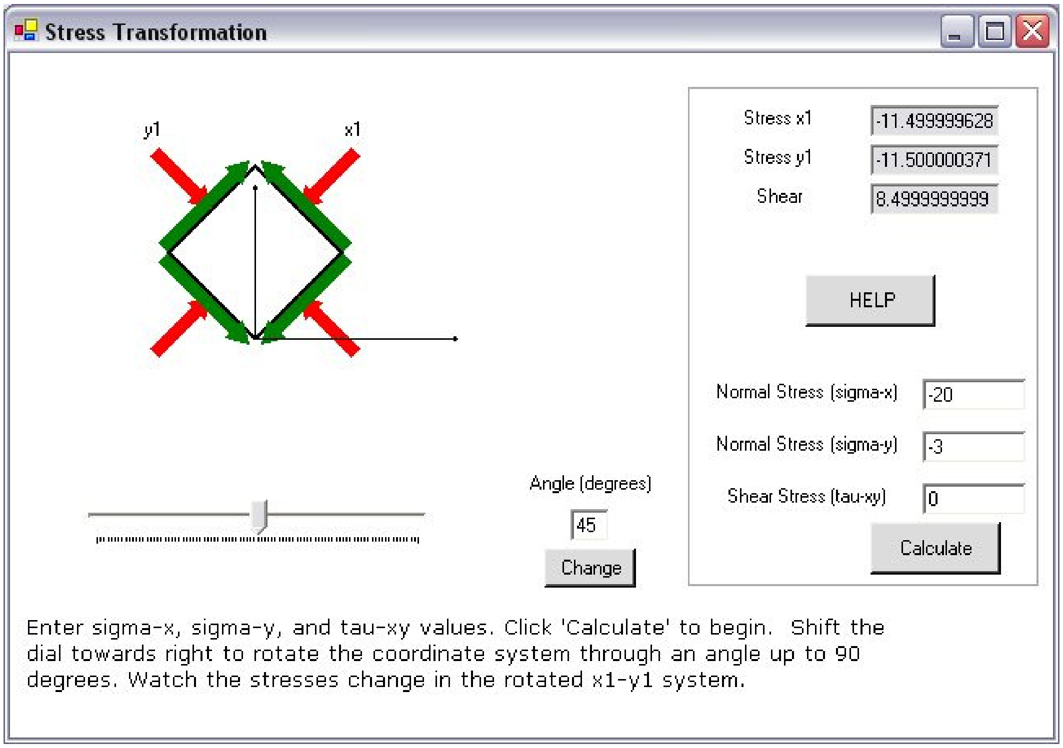


Figure-13

*Part-b*

Imagine that n=x and y=t. When the xy-system (nt-system) is turned through an angle of 28°, the x1-axis becomes aligned with the p=axis and y1-axis becomes aligned with the q-axis.

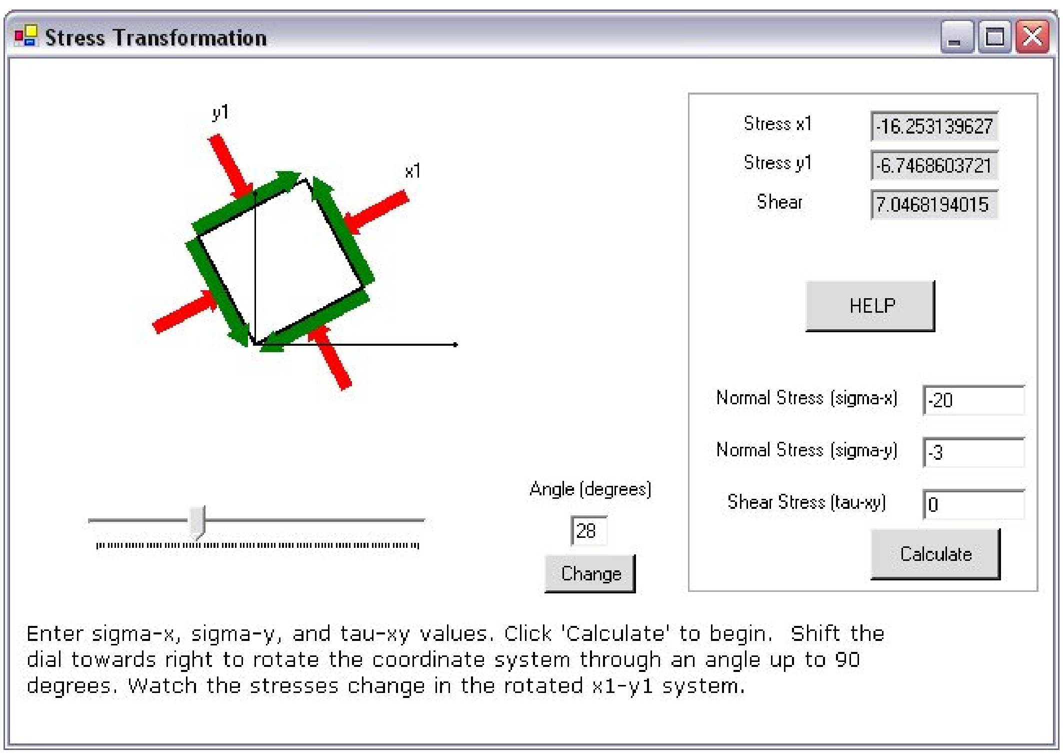


Figure-14

From Fig.(14)

$$\sigma_p = -16.3ksi \quad \sigma_q = -6.75ksi \quad \tau_{pq} = 7.05ksi$$