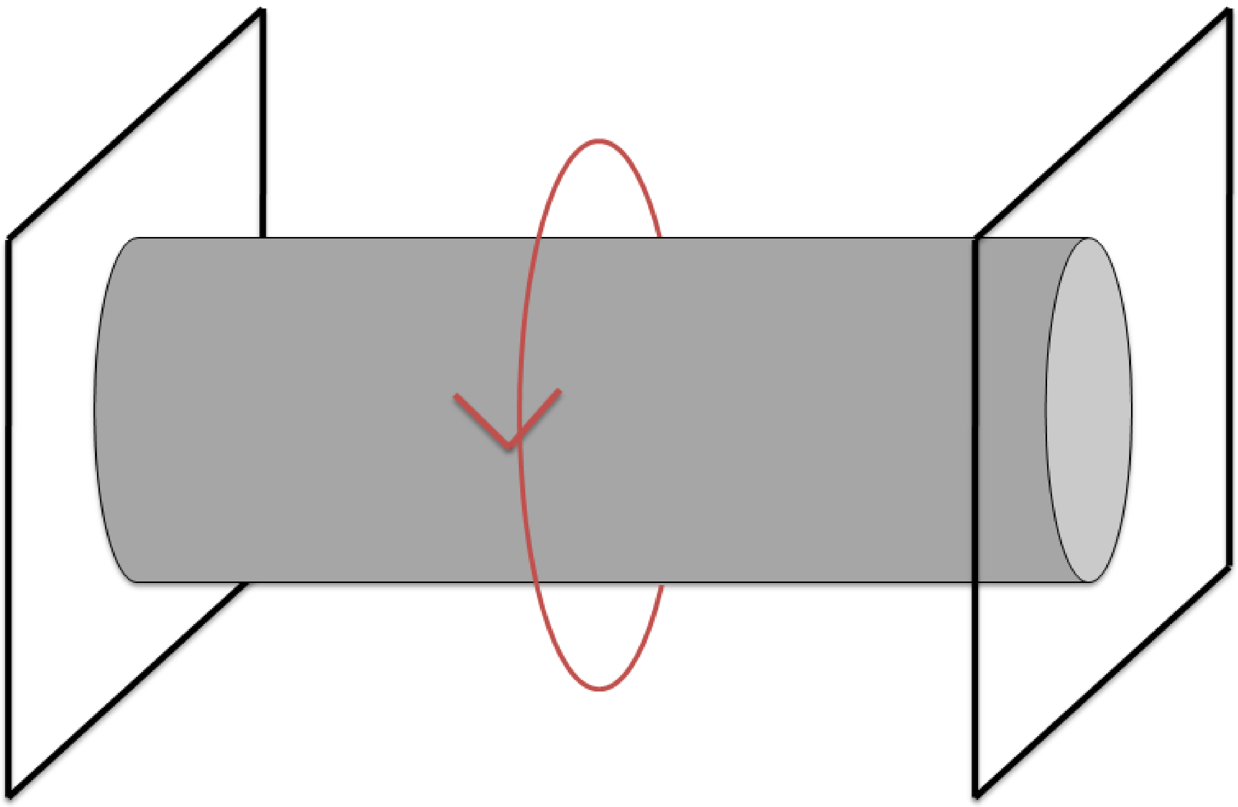
Example Problem:

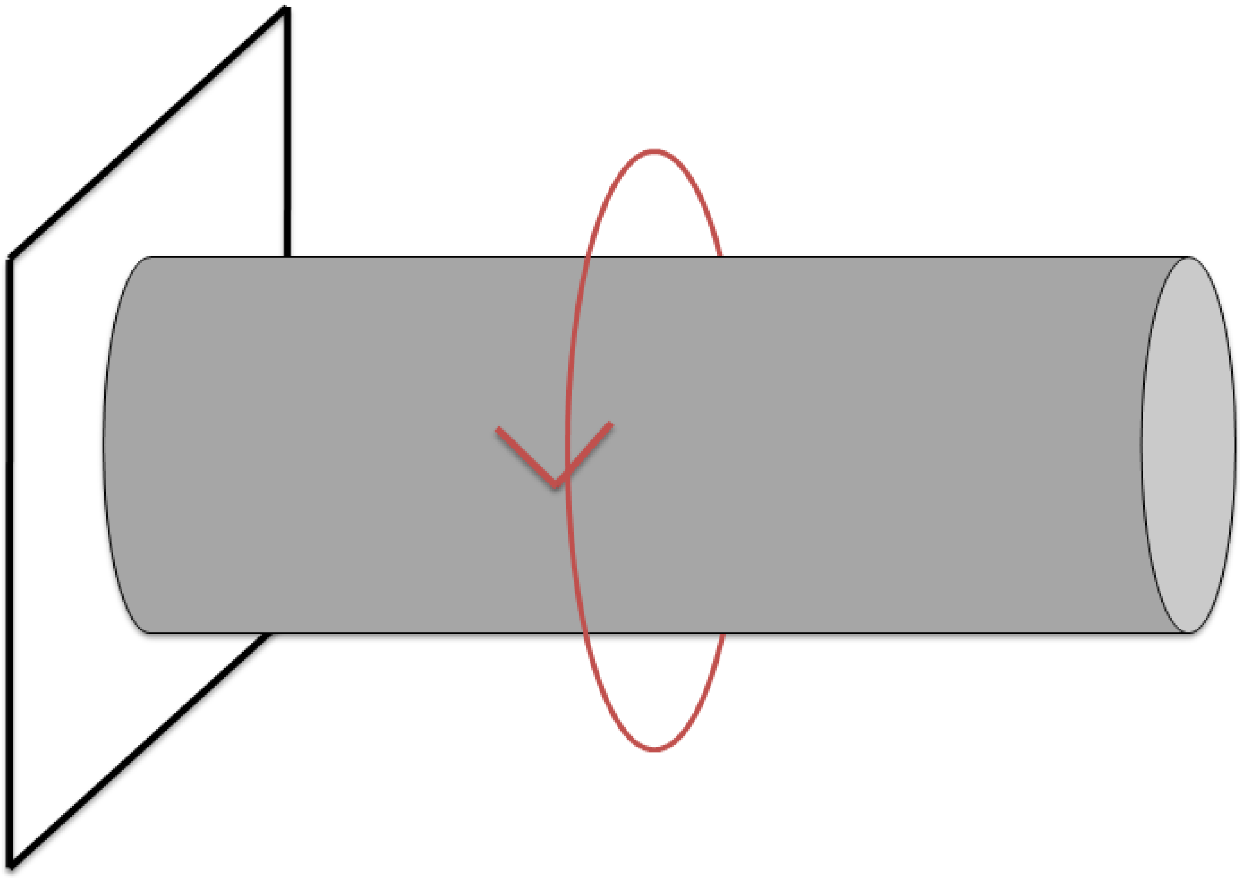
A uniform bar (L=60in) is clamped at each end. A load P=2000in-lb is applied to the right at a point x=30in from the left clamp. The bar is made of steel (G=11.5e6 psi) and has a diameter of 2 in.



Solution:

In order to solve the indeterminate problem, we will remove the right clamp and replace it with a resultant torque T_r. In this method, the indeterminate problem can be solved with a series of simple determinate problems.

First, remove the right clamp and solve as determinate problem



Segment 1: x1=0in x2=30in

Segment 2: x1 = 30in x2 = 60in

In segment 2, the torque is 0lb. Both the stress and twist in this section are zero.

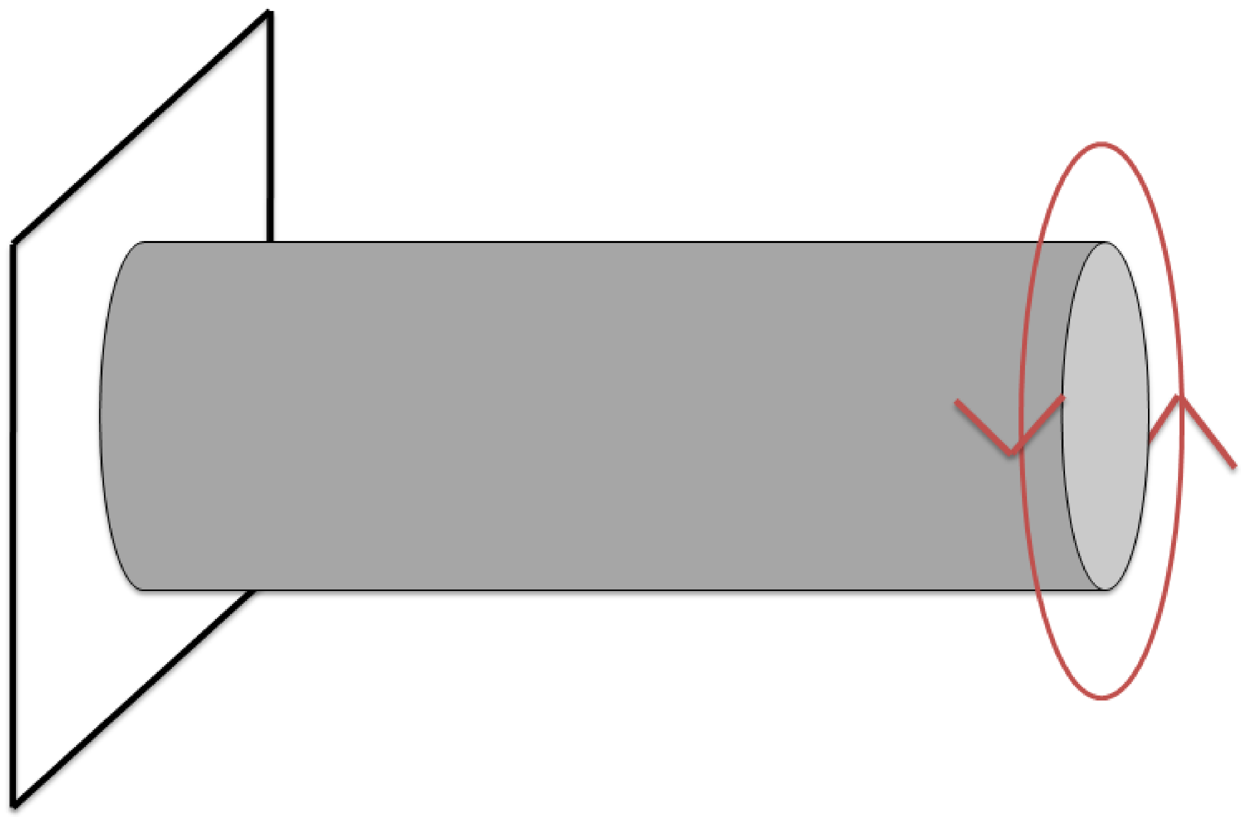
Segment 1:

$$J = \frac{\pi d^4}{32} = \frac{\pi * (2in)^4}{32} = 1.57in^4$$

$$\tau_{free} = \frac{.5Td}{J} = \frac{.5 * 2000in \cdot lb * 2in}{1.57in^4} = 1273.24\, psi$$

$$\theta_{free} = \frac{TL}{JG} = \frac{2000\ in \cdot lb\ 30\ in}{1.57 in^4 * 11.5e6\ \text{psi}} = 3.321\text{e-}3\ \text{rad}$$

Next, solve as determinate problem with same geometry, and unit torque at free right end.



Segment 1: x1=0in x2=60in

$$\tau_{unit} = \frac{.5Td}{J} = \frac{.5 * 1in \cdot lb * 2in}{1.57in^4} = 0.637\ psi$$

$$\theta_{unit} = \frac{TL}{JG} = \frac{1\ in \cdot lb\ 60\ in}{1.57 in^4 * 11.5e6\ \text{psi}} = 3.323\text{e-}6\ \text{rad}$$

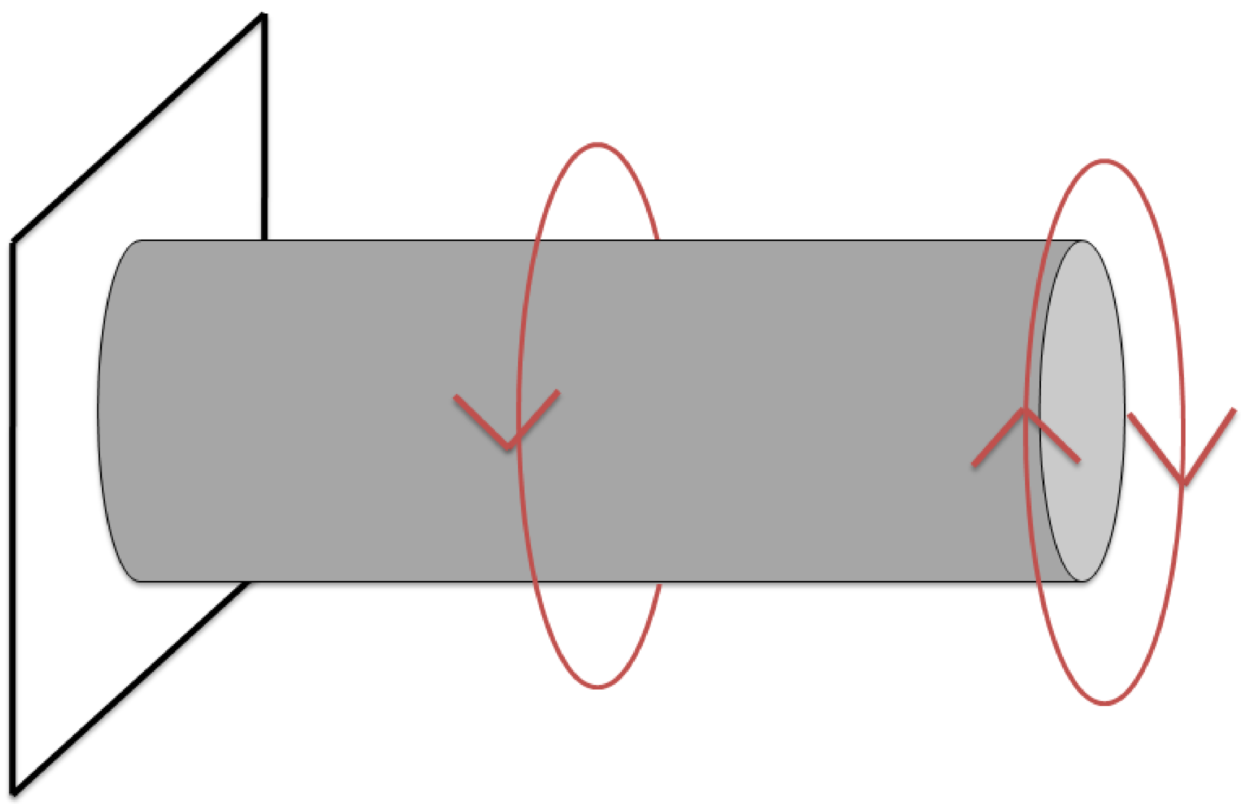
This value will be multiplied by T_r in order to find the “twist” caused by the resultant torque on the right end.

Now, we combine these results. We can see that, because the bar is clamped on both ends, the net twist of the bar must be zero. That is:

$$\theta_{free} + T_r \theta_{unit} = 0$$

$$T_r = -\frac{\theta_{free}}{\theta_{unit}} = -\frac{3.321\text{e}^{-3}\text{ rad}}{3.323\text{e}^{-6}\text{ rad}} = -999.493 in \cdot lb$$

Finally, we solve the problem as a determinate bar with T_r acting on the right end.



Segment 1: x1=0in x2=30in

Segment 2: x1 = 30in x2 = 60in

Segment 2:

$$T_2 = T_r$$

$$\tau_2 = \frac{.5T_2 d}{J} = \frac{.5 * -999.493 in \cdot lb * 2in}{1.57 in^4} = -636.62\ psi$$

$$\theta_2 = \frac{T_2 L}{JG} = \frac{-999.493\ in \cdot lb\ 30\ in}{1.57 in^4 * 11.5e6\ \text{psi}} = -1.661e\text{-}3 \text{ rad}$$

Segment 1:

$$T_1 = -T_r + T = -999.493 + 2000 = 1000.51\ in \cdot lb$$

$$\tau_1 = \frac{.5T_1 d}{J} = \frac{.5 * 1000.51 in \cdot lb * 2in}{1.57 in^4} = 636.266\ psi$$

$$\theta_1 = \frac{T_1 L}{JG} = \frac{1000.51\ in \cdot lb\ 30\ in}{1.57 in^4 * 11.5e6\ \text{psi}} = 1.662e\text{-}3 \text{ rad}$$

We can see that $\theta_1 = -\theta_2$, so the total twist of the bar is zero.