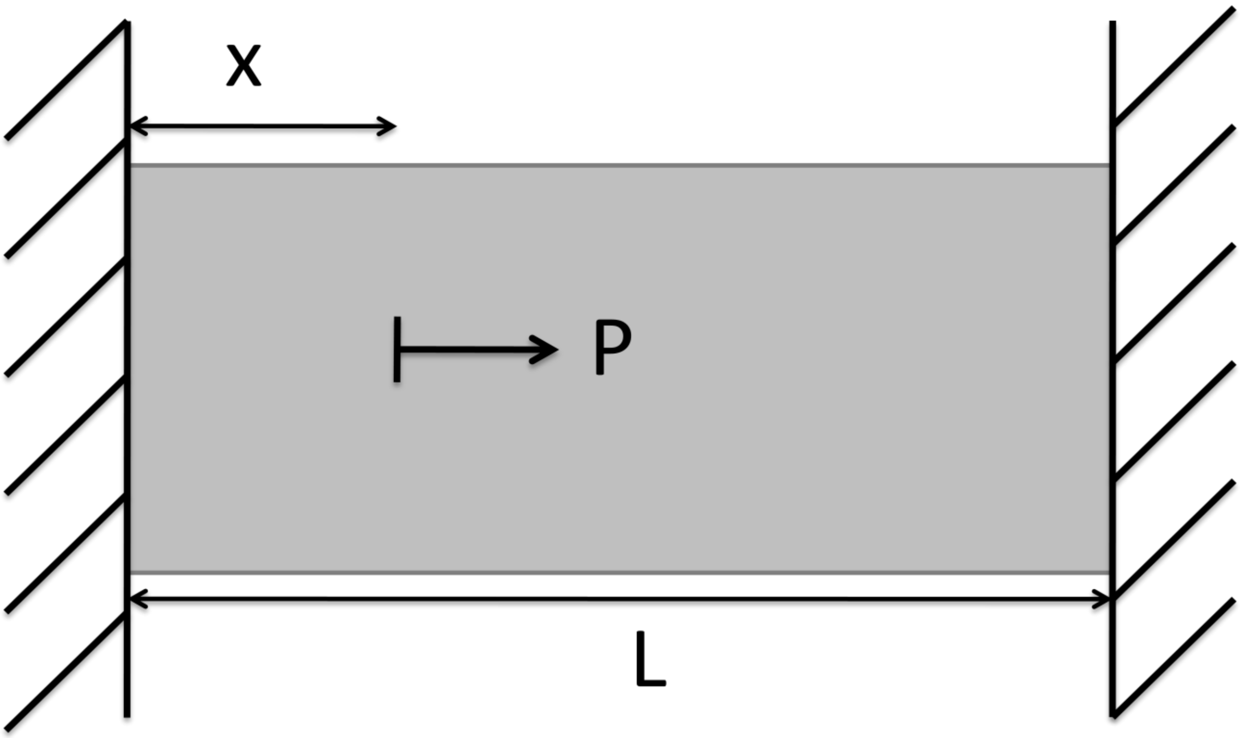
Example Problem:

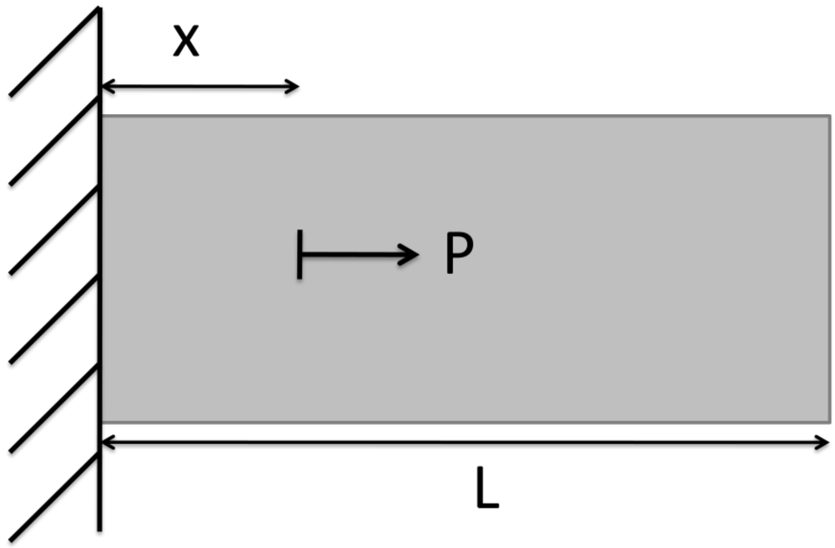
A uniform beam (L=100in) is clamped at each end. A load P=2000lb is applied to the right at a point x=30in from the left clamp. The beam is made of steel (E=30e6 psi) and has a cross-sectional area of 20 in^2.



Solution:

In order to solve the indeterminate problem, we will remove the right clamp and replace it with a resultant force P_r. In this method, the indeterminate problem can be solved with a series of simple determinate problems.

First, remove the right clamp and solve as determinate problem



Segment 1: x1=0in x2=30in

Segment 2: x1 = 30in x2 = 100in

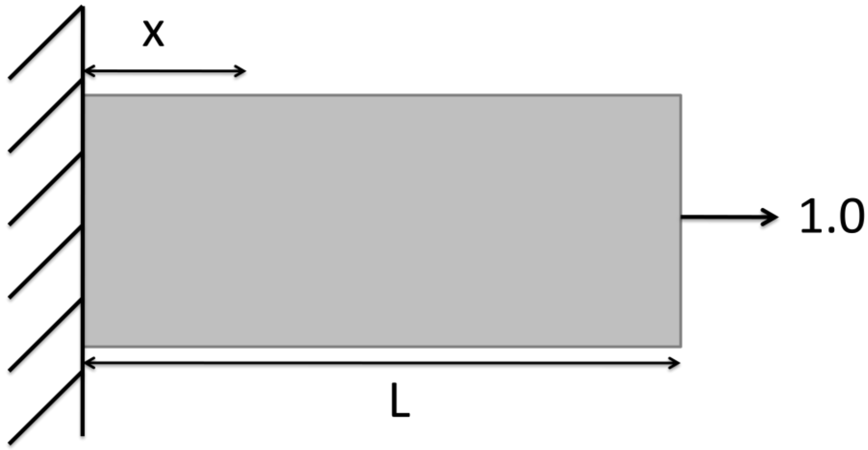
In segment 2, the load is 0lb. Both the stress and displacement in this section is zero.

Segment 1:

$$\sigma_{free} = \frac{P}{A} = \frac{2000\ lb}{20\ in^2} = 100\ psi$$

$$\delta_{free} = \frac{PL}{AE} = \frac{2000\ lb\ 30\ in}{20\ in^2\ 30e6\ psi} = 1.0e\text{-}4\ in$$

Next, solve as determinate problem with same geometry, and unit load at free right end.



Segment 1: x1=0in x2=100in

$$\sigma_{unit} = \frac{P}{A} = \frac{1\ lb}{20\ in^2} = 0.05\ psi$$

$$\delta_{free} = \frac{PL}{AE} = \frac{1\ lb\ 100\ in}{20\ in^2\ 30e6\ psi} = 1.667\text{e-}7\ in$$

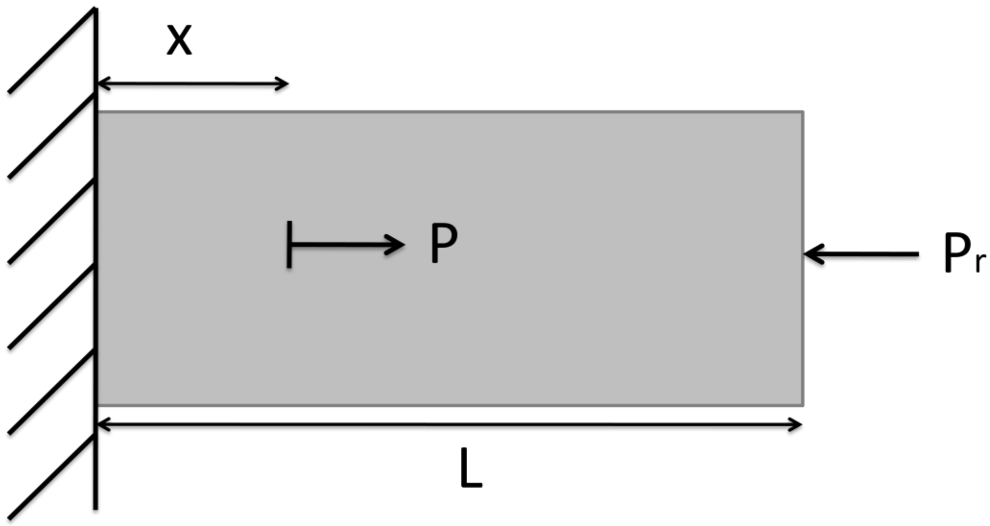
This value will be multiplied by P_r in order to find the "deflection" caused by the resultant load on the right end.

Now, we combine these results. We can see that, because the beam is clamped on both ends, the net deflection of the beam must be zero. That is:

$$\delta_{free} + P_r \delta_{unit} = 0$$

$$P_r = -\frac{\delta_{free}}{\delta_{unit}} = -\frac{1.0\text{e} - 4\ in}{1.667\text{e} - 7\ in} = -600 lb$$

Finally, we solve the problem as a determinate beam with P_r acting on the right end.



Segment 1: x1=0in x2=30in

Segment 2: x1 = 30in x2 = 100in

Segment 2:

$$P_2 = -P_r$$

$$\sigma_2 = \frac{P_2}{A} = \frac{-600\ lb}{20\ in^2} = -30\ psi$$

$$\delta_2 = \frac{P_2 L_2}{AE} = \frac{-600\ lb\ 70\ in}{20\ in^2\ 30e6\ psi} = -7.0\text{e-}5\ in$$

Segment 1:

$$P_1 = -P_r + P = -600 + 2000 = 1400 lb$$

$$\sigma_1 = \frac{P_1}{A} = \frac{1400\ lb}{20\ in^2} = 70\ psi$$

$$\delta_1 = \frac{P_1 L_1}{AE} = \frac{1400\ lb\ 30\ in}{20\ in^2\ 30e6\ psi} = 7.0\text{e-}5\ in$$

We can see that $\delta_1 = -\delta_2$, so the total deflection of the beam is zero.