Lecture - 9

Ambar K. Mitra

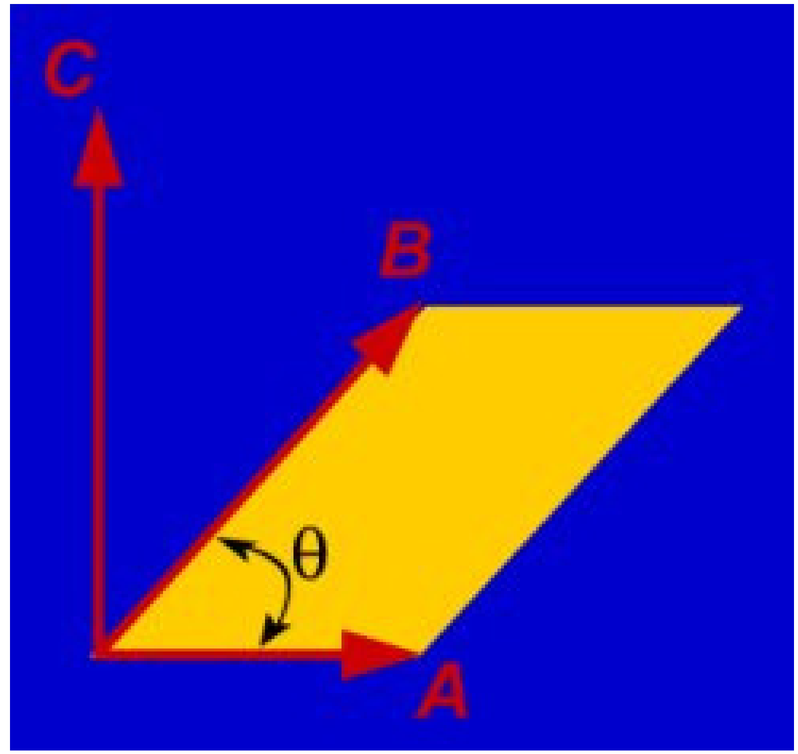
Vector Cross Product

$$\vec{C} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix}$$

$$C_x = (A_y B_z - A_z B_y)$$

$$C_y = (A_z B_x - A_x B_z)$$

$$C_z = (A_x B_y - A_y B_x)$$



Magnitude of *C* = Area of Parallelogram
Direction of *C* is perpendicular to *A* and *B*

$$|C| = |A||B|\sin\theta$$

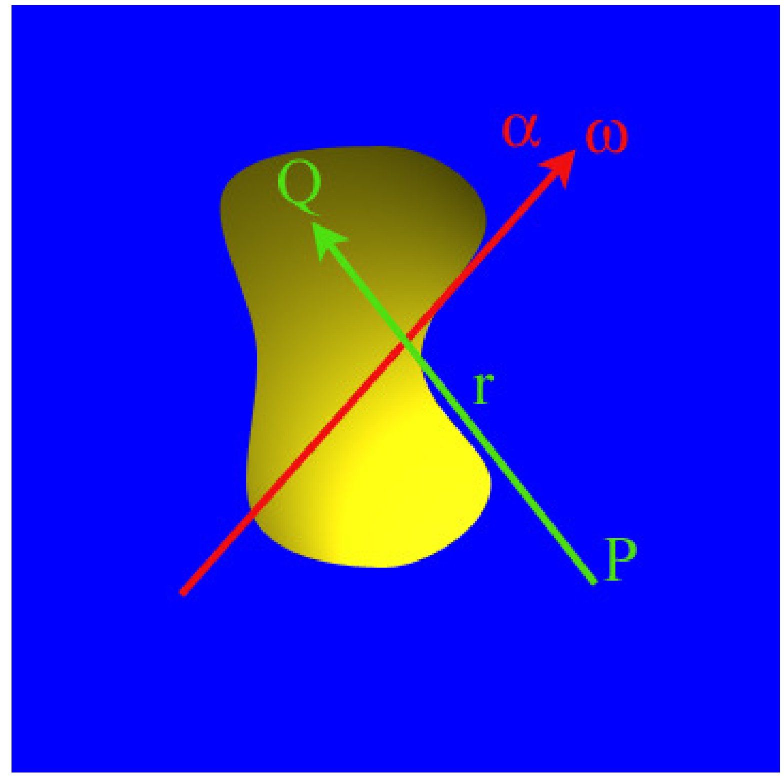
Exercise

Find the cross product of *A (7, -3, 11)* and *B (2, 3, -2)*.

Find the sin of the angle between *A* and *B*.

Angular Velocity & Acceleration

A body is spinning with angular velocity ω and angular acceleration α.



Q is a point on the body. P is an observer. Velocity and acceleration of Q as seen by P are:

$$\vec{v} = \vec{\omega} \times \vec{r} \qquad \vec{a} = \vec{\alpha} \times \vec{r}$$

Exercise

A body has angular velocity (3, -4, 5) and angular acceleration (2, 0, -4).

An observer is at location P(12, 10, -5).
A point on the body is at location Q(8, -5, 3).

The r-vector has tail at P and tip at Q.

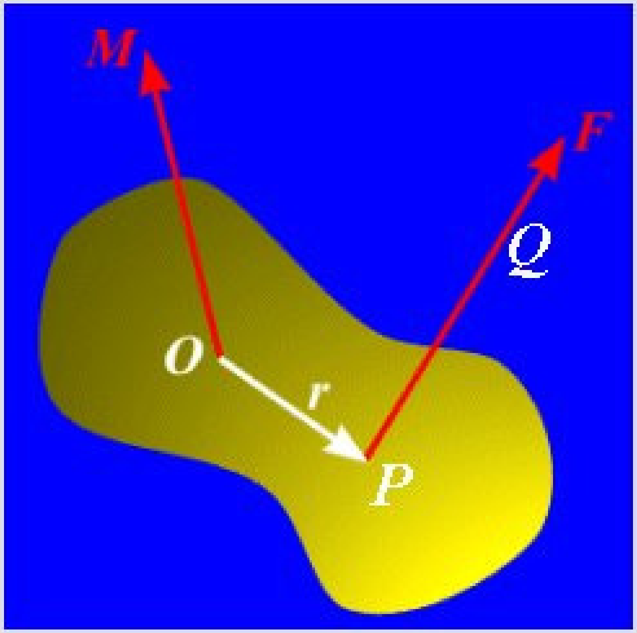
Find velocity and acceleration of Q as seen by P.

Moment at a Point

A force is applied on a body at point P.

Moment at point O is the tendency of rotation due to the force at P.

$$\vec{M} = \vec{r} \times \vec{F}$$



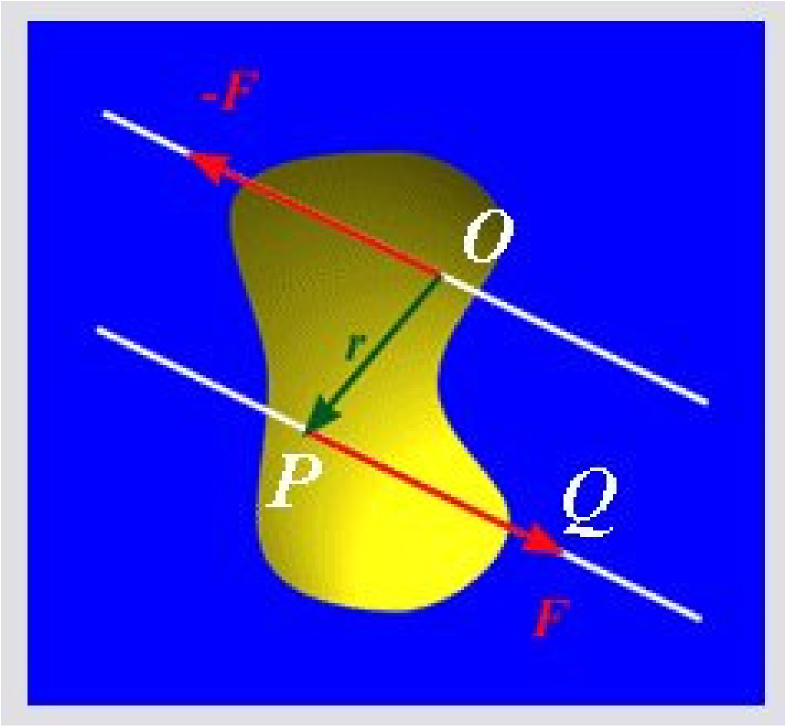
Exercise

A force F (200, 100, -200) is applied at (5, -6, 7). Find the moment at (1, 1, 1).

Couple

Two equal and opposite forces produce a tendency of “pure spin” in a body.

$$\vec{C} = \vec{r} \times \vec{F}$$



Exercise

A force (-)F (12, -10, 14) is applied at (0, 2, 3). Another force (+)F (-12, 10, -14) is applied at (7, 9, 11). Find the couple.

Remember that the r-vector has its tail on (-)F and the tip on (+)F.

Moment about a Hinge Line

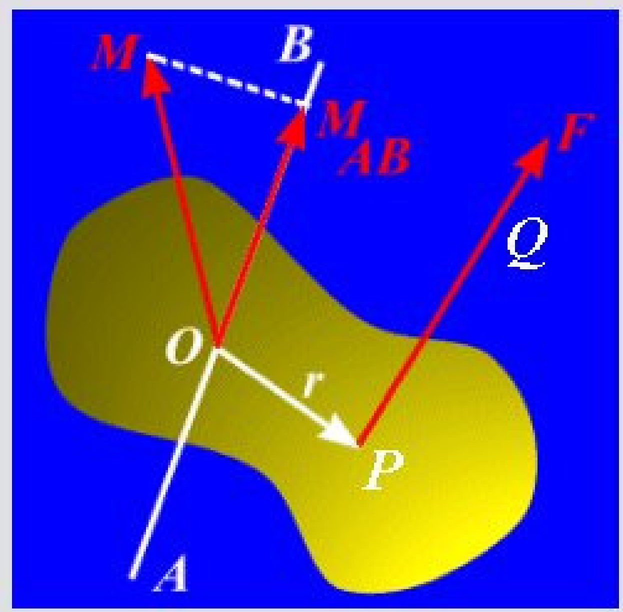
Calculation of moment about a hinge is a two step process involving cross and dot product.

$$\vec{M}_O = \vec{r} \times \vec{F} \; ; \; \hat{n}_{OB} = \frac{\vec{OB}}{|OB|}$$

$$M_{AB} = \vec{M}_O \cdot \hat{n}_{OB}$$

$$\vec{M}_{AB} = M_{AB} \hat{n}_{OB}$$

Force applied at P.
Hinge is along the line OB.



Although the net moment is M, but the tendency of rotation is due to M_{AB}.

Exercise

A force F(12, 10, -5) is applied at (1,1,1).

Find the moment about the hinge along the line between points

O(2, 3, 4) and B(5, -2, 4).