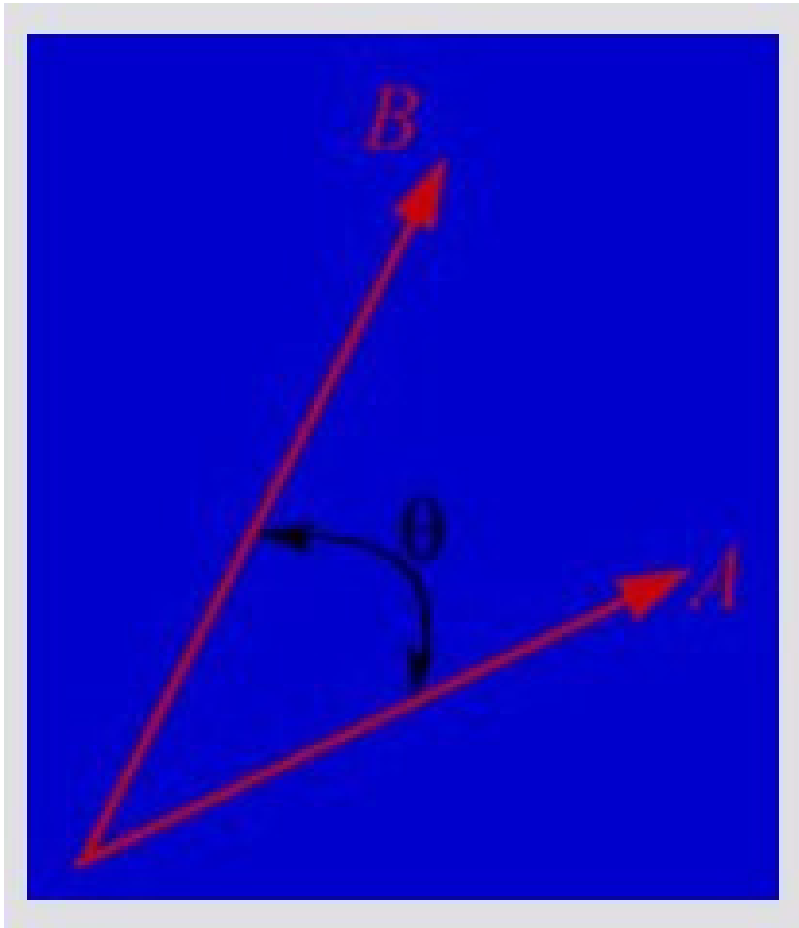


Lecture - 8

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Vector Dot Product



$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

Dot Product Example

1. Find the dot product of the two vectors $(3,7,-9)$ and $(5,6,8)$.
2. Find the cosine of the angle between the two vectors $(3,7,-9)$ and $(5,6,8)$.

Components

- A force F is applied on a rod with end points at $A(a,b,c)$ and $P(p,q,r)$.
- We want to break this force into two components (parts); one parallel to the rod and one perpendicular to the rod.
- The parallel component stretches the rod and the perpendicular component bends the rod.

Dot Product and Components

$$\vec{N} = (p - a)\hat{i} + (q - b)\hat{j} + (r - c)\hat{k}$$

$$|\vec{N}| = \sqrt{(p - a)^2 + (q - b)^2 + (r - c)^2}$$

$$\hat{n} = \frac{\vec{N}}{|\vec{N}|}$$

$$\vec{F}_{\text{par}} = (\vec{F} \cdot \hat{n})\hat{n}$$

$$\vec{F}_{\text{per}} = \vec{F} - \vec{F}_{\text{par}}$$

Example

1. A force $(200, 100, -300)$ is applied on a rod with end points at $(1, 2, 3)$ and $(5, 6, -8)$. Find the components parallel and perpendicular to the rod.

Force Perpendicular to a Plane

A force F is applied to a plane

$$Ax + By + Cz + D = 0$$

The plane could be the tail-gate of your pick-up truck.

We want to find the components of the force parallel and perpendicular to the plane.

Dot Product and Components

$$\vec{N} = A\hat{i} + B\hat{j} + C\hat{k}$$

$$|\vec{N}| = \sqrt{A^2 + B^2 + C^2}$$

$$\hat{n} = \frac{\vec{N}}{|\vec{N}|}$$

$$\vec{F}_{\text{par}} = (\vec{F} \cdot \hat{n})\hat{n}$$

$$\vec{F}_{\text{per}} = \vec{F} - \vec{F}_{\text{par}}$$

Example

A $(200, 100, -300)$ force is applied to the plane $2x - 3y + 4z + 5 = 0$. Find the components parallel and perpendicular to the plane.