Lecture - 5

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Integration by Substitution

For integrals

$$\int f(g(x))\frac{dg}{dx}dx$$

Substitute

$$z = g(x)$$

$$dz = \frac{dg}{dx}dx$$

Then integral becomes

$$\int f(z)dz$$

Substitute $z = 1 + x^2$

$$\int (1 + x^2)^k x dx$$

$$\int \sin(1 + x^2) x dx$$

$$\int e^{(1+x^2)} x dx$$

$$\int \mathrm{Ln}(1 + x^2) x dx$$

Substitute $z = 1 + x^3$

$$\int (1 + x^3)^k x^2 dx$$

$$\int \sin(1 + x^3) x^2 dx$$

$$\int e^{(1+x^3)} x^2 dx$$

$$\int \mathrm{Ln}(1 + x^3) x^2 dx$$

Exercise

$$\int \frac{dx}{\sqrt{x}\left(1+\sqrt{x}\right)^3}$$

$$\int \frac{\left[1+\frac{1}{x^2}\right]^{5/2}}{x^3}dx$$

$$\int \frac{1}{\sqrt{x-2}}dx$$

$$\int \frac{x^2+1}{\sqrt{x^3+3x}}dx$$

$$\int \frac{x^3}{(1+x^4)^{1/3}}dx$$

$$\int \frac{\sin(\mathrm{L}nx)}{x}dx$$

Exercise

$$\int \frac{1}{xLnx}\,dx$$

$$\int \frac{\cos(5x)}{e^{\sin(5x)}}\,dx$$

Exercise

$$\int (x+3)\sqrt{x-1}\,dx$$

$$z = x - 1$$

$$x + 3 = z + 4$$

$$\int (z+4)\sqrt{z}\,dz = \int z^{3/2}\,dz + \int 4\sqrt{z}\,dz$$

Exercise

$$\int \frac{x+5}{2x+3}\,dx$$

$$z = 2x+3$$

$$x+5 = \frac{1}{2}z + \frac{7}{2}$$

$$\int \left(\frac{1}{2} + \frac{7}{2}\frac{1}{z}\right)\frac{1}{2}dz = \int \left(\frac{1}{4} + \frac{7}{4}\frac{1}{z}\right)dz = \frac{1}{4}z + \frac{7}{4}\,Lnz$$

$$= \frac{1}{4}(2x+3) + \frac{7}{4}\,Ln(2x+3)$$