# Lecture-2

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# Function of a Function

The concept is simple: $\sin^2 x = (\sin x)^2$

To evaluate this, first you calculate sin x, then take the result and square it.

The first step (finding sin x) is the inner layer.

The second step (squaring) is the outer layer.

# Math Notation

In math notation we write $y=\sin^2 x$ as follows

$$y = f\{ z(x) \}$$

where

$$z(x) = \sin x$$

and

$$f\{z\} = z^2$$

# Practice

Write $e^{2x}$ in the $f\{z(x)\}$ form

$$z(x)= 2x \; ; \; f\{z\} = e^z$$

Write sin(cos x) in the $f\{z(x)\}$ form

$$z(x)=\cos x \; ; \; f\{z\} = \sin z$$

Write cos(2x + 1) in the $f\{z(x)\}$ form

$$z(x)=2x + 1 \; ; \; f\{z\} = \cos z$$

# Function of a Function of a Function of a Function... and so on

You can keep adding layers to build an onion.

The outermost layer of the onion is what you did last to calculate the value of the formula on your calculator

$$f\,[z\{w(x)\}]$$

$$w(x) = 2x + 1$$

$$z(w) = w^{-3}$$

$$f(z) = \sin z$$

Apply the technique on

$$\sin\left(\frac{1}{2x+1}\right)^3$$

# Practice

$$e^{(2x-7)^2}$$

$$w(x) = 2x - 7$$
$$z(w) = w^2$$
$$f(z) = e^z$$

$$w(x) = 2x + 4$$
$$z(w) = \sin w$$
$$f(z) = \ln z$$

$$\ln\{\sin(2x + 4)\}$$

# Chain Rule

$$y = f\{ z(x) \}$$

$$\frac{dy}{dx} = \frac{df}{dz}\frac{dz}{dx}$$

Imagine that derivative is a knife, it cuts through the outermost layer of the onion first.

$$y = f[ z \{ w(x) \} ]$$

$$\frac{dy}{dx} = \frac{df}{dz}\frac{dz}{dw}\frac{dw}{dx}$$

# Practice

$$f(x) = e^{(2x-7)^2}$$

$$w(x) = 2x - 7; \quad \frac{dw}{dx} = 2$$

$$z(w) = w^2; \quad \frac{dz}{dw} = 2w = 2(2x-7)$$

$$f(z) = e^z; \quad \frac{df}{dz} = e^z = e^{w^2} = e^{(2x-7)^2}$$

$$\frac{df}{dx} = \frac{df}{dz}\frac{dz}{dw}\frac{dw}{dx} = e^{(2x-7)^2}\, 2(2x-7)2$$

$$= 4(2x-7)e^{(2x-7)^2}$$

# Practice

$$f(x) = ln\{sin(2x+4)\}$$

$$w(x) = 2x+4; \quad \frac{dw}{dx} = 2$$

$$z(w) = sin\,w; \quad \frac{dz}{dw} = cos\,w = cos(2x+4)$$

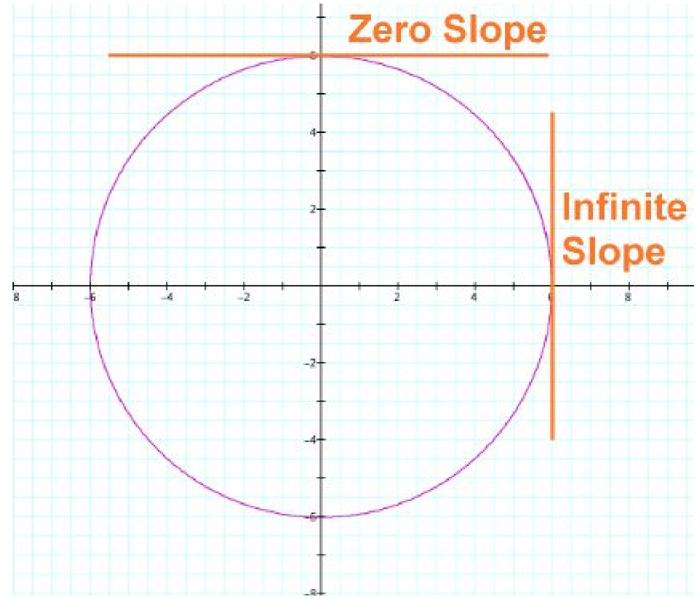
$$f(z) = ln\,z; \quad \frac{df}{dz} = \frac{1}{z} = \frac{1}{sin\,w} = \frac{1}{sin(2x+4)}$$

$$\frac{df}{dx} = \frac{df}{dz}\frac{dz}{dw}\frac{dw}{dx} = \frac{1}{sin(2x+4)}cos(2x+4)2$$

$$= 2cot(2x+4)$$

# Chain Rule for Intrinsic Functions

$$x^2 + y^2 = 6; \quad Find \frac{dy}{dx}$$




$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0$$

$$2x + \frac{d}{dy}(y^2)\frac{dy}{dx} = 0$$

$$2x + 2y\frac{dy}{dx} = 0; \quad 2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

**At y=0, the slope of the tangent is infinite.**

**Infinite slope is a vertical line.**

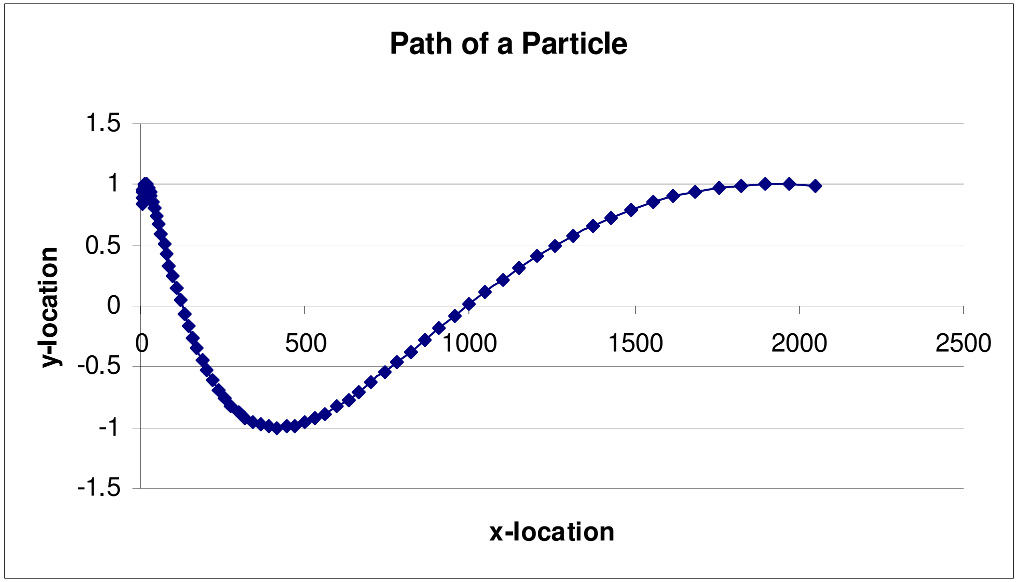
**Zero slope is a horizontal line.**

# Parametric Derivative

$$x = 4t^3; \quad y = \sin t; \quad Find\ \frac{dy}{dx}$$

$$\frac{dx}{dt} = 12t^2; \quad \frac{dy}{dt} = \cos t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{12t^2}$$




**dy/dx is the slope of the path. Shows the direction of velocity at an instant.**